Homework 14 (the last homework[1])

Math 469 (section 500), Spring 2019

This homework is due on Thursday, April 25.

0. Read (or remind yourself) what *Descartes's rule of signs* says (page 24 of your textbook, or find it online).

1. Consider the polynomial $f(x) = x^3 + ax + b$, where $a, b \in \mathbb{R}$.

   (a) What is the maximum number of positive roots that f can have? Explain. (A *positive root* of f is some $x^* > 0$ for which $f(x^*) = 0$).

   (b) Give an example of $a, b \in \mathbb{R}$ so that f has *fewer* positive roots than the maximum number you found in (a). Justify your answer.

   (c) Give an example of $a, b \in \mathbb{R}$ so that f has exactly the maximum number of positive roots you found in (a). Justify your answer.

2. This problem concerns the article, *The core control system of intracellular iron homeostasis: A mathematical model*, by Chifman *et al.* (2012), available at `https://doi.org/10.1016/j.jtbi.2012.01.024`.

   (a) State one scientific question addressed in the article.

   (b) State one mathematical question addressed in the article.

   (c) Does this article involve *forward* or *backward* modeling? Explain.

   (d) Examine the 5 ODEs underlying their model (in Section 3.2). Is there a network for which the mass-action ODEs are those 5 ODEs? Explain.

   (e) What do the authors claim about the number of equilibria and their stability? Do these depend on the parameters (α_i, γ_i, k_{ij}, etc.)?

   (f) Is your answer to (e) consistent with the biology (according to the article)?

3. Consider the following iron-trafficking model, which may be viewed as a much-simplified version of the Chifman *et al.* (2012) model:

$$\frac{dC}{dt} = k_1 \left(\frac{1}{1 + (\frac{C}{T})^n} \right) - k_2 C \left(1 - \frac{1}{1 + (\frac{C}{T})^n} \right) - k_3 C - \alpha C$$

$$\frac{dF}{dt} = k_2 C \left(1 - \frac{1}{1 + (\frac{C}{T})^n} \right) - \alpha F \ ,$$

   where C represents iron in the cytosol and F represents ferritin, and n is a positive integer, $k_i > 0$ for all i, $\alpha > 0$, $T > 0$.

   (a) How does the number of equilibria $(C, F) \in \mathbb{R}^2_{>0}$ depend on the values of n, k_i, T, α?

   (b) For each equilibria, is it locally stable? Does the stability depend on n, k_i, T, α?

   (c) *(Optional bonus problem)* Is this system globally stable?

[1] we will continue to do in-class peer evaluations