Fall 2005 Math 151

Night Before Drill

courtesy: Amy Austin

Review Exercises: Sections 4.4 - 5.7

Section 4.4

1. Find $f'(x)$ for $f(x) = \ln(2x^2 - 8)$
2. Find y' for $y = (\cos x)^{\tan x}$
3. Find $f''(e)$ for $f(x) = \ln(\ln x)$

Section 4.5

4. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.

   a.) What is the half life of the substance?

   b.) How much of the substance remains after 2.5 years?

5. Suppose that a colony of bacteria is growing exponentially. If 12 hours are required for the number of bacteria to grow from 4000 to 6000, find the doubling time.
6. A bowl of soup at temperature 180° is placed in a 70° room. If the temperature of the soup is 150° after 2 minutes, when will the soup be an eatable 100°?
7. A tank contains 1000 L of brine with a concentration of .10 kg of salt per liter. Pure water enters the tank at a rate of 20 liters per minute. The well mixed solution exits the tank at the same rate. How much salt is in the tank after 35 minutes?

Section 4.6

8. Find $\tan(\sin^{-1}\frac{2}{5})$
9. Find the derivative of $y = x^2\cos^{-1}(e^{3x})$
10. Find the equation of the line tangent to $y = \tan^{-1}(2x-1)$ when $x = 1$.
11. Compute the exact value of $\lim\limits_{x\to\infty} \arccos\left(\frac{1+2x}{5-4x}\right)$
12. Compute $\sec(\arctan(-\sqrt{5}))$
13. Compute $\sin^{-1}(\sin\frac{4\pi}{3})$
14. $\tan(\arccos x)$ is equivalent to what?
15. Find the domian of $arcsin(1-8x^3)$

Section 4.8

16. Find the limits of each of the following:

    a) $\lim\limits_{x\to 0}\frac{\arcsin(3x)}{2x}$

    b) $\lim\limits_{x\to 0^+} 2x\cot x$

    c) $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x}$

    d.) $\lim\limits_{x\to 0^+}\left(\frac{1}{x}\right)^{\sqrt{x}}$

    e.) $\lim\limits_{x\to\infty}\frac{(\ln x)^2}{4x}$

Section 5.1 - 5.3

17. If $f(x) = \frac{1}{x}$, verify $f(x)$ satisfies the Mean Value Theorem on the interval $[1, 10]$ and find all c that satisfies the conclusion of the Mean Value Theorem.
18. Find the absolute maximum and minimum of the given function on the given interval.

    a) $x^3 - 5x^2 + 3$ on $[-1, 3]$

    b) $x\ln x$ on $[e^{-2}, 1]$

19. Find the intervals where the given function is increasing and decreasing and identify all local extrema:

    a) $f(x) = x^3 - 2x^2 + x$

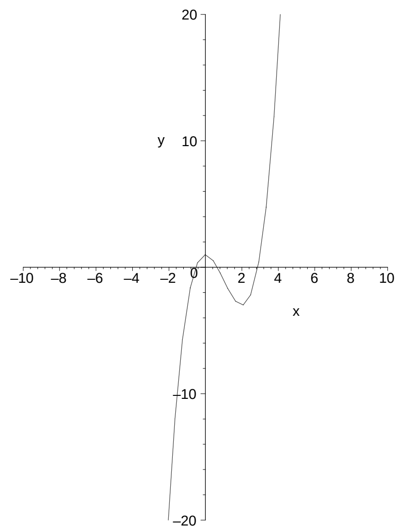
    b) $f(x) = x^2e^{2x}$

    c) $f(x) = \sin x + x$, $0 \le x \le 2\pi$

    d.) $f(x) = \frac{x}{(x-1)^2}$

20. Determine where the graph of $f(x) = x^4 + 8x^3 + 13$ is concave up and concave down and find the inflection points.

21. In the graph that follows, the graph of f' is given. Using the graph of f', determine all critical values of f, where f is increasing and decreasing, local extrema of f, where f is concave up and concave down, and the x-coordinates of the inflection points of f. Assume f is continuous.



Section 5.5

22. A cardboard box holding 32 cubic inches with a square base and open top is to be constructed. Find the minimum area of cardboard needed.

23. Find the shortest distance from the point $(1, 4)$ to the parabola $y^2 = 2x$

24. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius 2.

Section 5.7

25. Given $f''(x) = 2e^x - 4\sin(x)$, $f(0) = 1$, and $f'(0) = 2$, find $f(x)$.

26. A particle accelerates according to the equation $a(t) = .12t^2 + 4$. If the initial velocity is 10 and the initial position is 0, find the position function $s(t)$.

27. A stone is dropped from a 450 meter tall building.

    a.) Find a formula for the height of the stone at time t.

    b.) With what velocity does the stone hit the ground?

28. A car is traveling at a speed of 220/3 feet per second when the brakes are fully applied thus producing a constant deceleration of 40 feet per second squared. How far does the car travel before coming to a stop?

29. A pie is thrown vertically upward from a height of 4 feet with a speed of 25 feet per second. What is the maximum height the pie attains?

30. Find the vector functions that describe the velocity and position of a particle that has an acceleration of $\mathbf{a}(t) = \langle 0, 2\rangle$, initial velocity of $\mathbf{v}(0) = \langle 1, -1\rangle$ and an initial position of $\mathbf{r}(0) = \langle 0, 0\rangle$.