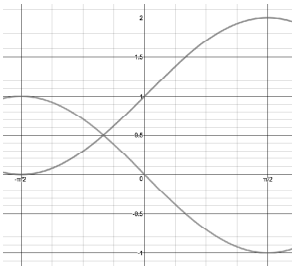
Math 152 Week-in-Review

Final Exam Review Monday, May 1, 1-3pm HELD 100

1. Evaluate $\displaystyle\int_0^{(\pi^2/9)} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$.
2. Find the area of the region bounded by $y = x^2 - 2x$ and $y = 2x - 3$.
3. Write an integral for the area of the region bounded by $y = -\sin x$ and $y = 1 + \sin x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$. Do not evaluate.



4. Consider the region bounded by $y = x^3$, $x = 0$, $y = 8$ and $y = 1$. Find the volume if this region is revolved around the y-axis.
5. Consider the region bounded by $y = 4 - x^2$ and $y = -2x + 1$. Set up an integral, but do not evaluate, for the volume if the region is rotated around the following lines.
   (a) $x = -2$
   (b) $y = 4$
6. The solid S has a base bounded by the functions $y = x^2$ and $y = \sqrt{x}$. Cross-sections perpendicular to the y-axis are squares. Set up an integral for the volume of S, but do not evaluate.
7. A spring has a natural length of 3 m. The force required to keep the spring stretched to a length of 8 m is 7 N. Find the work required to stretch the spring from a length of 4 m to a length of 6 m.
8. A rope that weighs $\frac{1}{6}$ pounds per foot is used to pull a bucket full of water to the top of a 60 ft well. The bucket weighs 15 pounds when full of water. How much work is done?
9. A tank has the shape of a cone, where the radius at the top is 2 m, and the cone is 7 m high. Suppose the water in the tank is 4 m deep, and all the water is pumped out of a spout 1 m above the top of the cone. Write an integral for how much work is done, but do not evaluate. Use ρg for the weight density of water.
10. Integrate the following
    (a) $\displaystyle\int_0^5 \frac{x}{\sqrt{9-x}}\,dx$
    (b) $\displaystyle\int \sqrt{x}\ln(x)\,dx$

(c) $\displaystyle\int_0^{\pi/6} \sin^2(2x)\,dx.$

(d) $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

(e) $\displaystyle\int \tan^3(x)\sec^5(x)\,dx$

(f) $\displaystyle\int_0^1 \frac{x^2}{e^x}\,dx$

(g) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-16}}$

(h) $\displaystyle\int_0^{1/3} \sqrt{4-9x^2}\,dx$

(i) $\displaystyle\int \frac{x^3-6x^2+7x-9}{x^2-3x}\,dx$

(j) $\displaystyle\int \frac{8x+12}{x^4+4x^2}\,dx$

(k) $\displaystyle\int_1^{\infty} 5x^2e^{-x^3}\,dx.$

(l) $\displaystyle\int_0^5 \frac{1}{2x-5}\,dx$

11. Determine if the following integral converges or diverges: $\displaystyle\int_3^{\infty} \frac{5-\cos(x)}{\sqrt{x-2}}\,dx$

12. Sketch the parametric curves described below.

    a.) $x = t^2$, $y = t + 5$

    b.) $x = 3\cos\theta$, $y = 2\sin\theta$, $0 \le \theta \le \pi$

13. What is the shape of the curve $x = -2 + \sin t$, $y = 5 + \cos t$? In what direction is the curve traced as t increases?

14. Find the length of the curve $x = 2t^2 + \dfrac{1}{t}$, $y = 8\sqrt{t}$, $1 \le t \le 3$.

15. Setup an integral, but do not evaluate, for the the surface area obtained by rotating the curve $x = 3t\cos\left(\frac{t}{2}\right)$ and $y = 2e^{t/3}$, $0 \le t \le \frac{\pi}{2}$, about the x-axis, then about the y-axis.

16. Write two polar representations, one with $r > 0$ and one with $r < 0$ for the point $(-5\sqrt{3}, 5)$.

17. Write a polar equation for the cartesian equation $x^2 + y^2 = 5x$. What is the shape of the graph?

18. Find a cartesian equation for $r = 2\csc\theta$. What is the shape of the graph?

19. Find the area inside the cardioid $r = 4 + 4\sin\theta$ that lies above the x-axis.

20. Find the area of the region inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.

21. Find the area of one petal of the rose $r = 2\cos(3\theta)$.

22. Determine if the sequence $a_n = \ln(2n + 3e^{-n}) - \ln(5n + 2)$ converges or diverges. If it converges, find its limit.

23. Determine if the sequence $a_n = \dfrac{(-1)^n(5n - n^3)}{2n^3 + 3n^2}$ converges or diverges. If it converges, find its limit.

24. Suppose that $s_n = \dfrac{2n-3}{n+4}$ is the sequence of partial sums for the series $\displaystyle\sum_{n=1}^{\infty} a_n$. Find a_4 and $\displaystyle\sum_{n=1}^{\infty} a_n$.

25. Given the recursive sequence $a_1 = 4$ and $a_{n+1} = 10 - \dfrac{21}{a_n}$ is bounded and increasing, find the limit.

26. Find a general formula for the sequence $\left\{-\frac{7}{4}, \frac{12}{9}, -\frac{17}{16}, \frac{22}{25}, -\frac{27}{36}, \ldots\right\}$. Assume the pattern continues and starts with $n = 1$.

27. Determine if the series $\displaystyle\sum_{n=1}^{\infty}\left[e^{1/n} - e^{1/(n+2)}\right]$ converges or diverges. If it converges, find its sum.

28. Determine if the series converges absolutely, converges but not absolutely, or diverges.
$$\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{\sqrt{n}}$$

29. Determine if the series converges or diverges and justify your answer.
$$\sum_{n=3}^{\infty} \frac{\cos^2(n) + 4}{n - 2}$$

30. Determine if the series converges or diverges. If it converges, find its sum
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n}{7^{n-1}}$$

31. Determine if the series converges absolutely, converges but not absolutely, or diverges.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^3 + 2n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (\sqrt{n} - 3)}{n + 7}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+2n)}{4n^2-3n}$

32. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n^2}}{n^3}$ converges or diverges.

33. Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 3^{2n}}{(-5)^{n+1}}$ converges or diverges.

34. Determine if the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\arctan n}{3+2e^{-n}}$$

35. Find the radius and interval of convergence for the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(2x-5)^n}{3^n \ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 7^{n-1}(x+3)^n}{n^4(n+2)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(4x-7)^n}{e^n\sqrt{n}}$

36. Suppose that the series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x=-4$ and diverges when $x=6$. What can be said about the convergence of the following series?

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n c_n 3^n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 4^n$ (c) $\displaystyle\sum_{n=0}^{\infty} c_n 5^n$ (d) $\displaystyle\sum_{n=0}^{\infty} c_n(-6)^n$ (e) $\displaystyle\sum_{n=0}^{\infty} c_n 9^n$

37. Write the Maclaurin series for the function $\dfrac{5x^4}{5+2x^2}$.

38. Write the Maclaurin series for the function $\ln(6-2x^3)$.

39. Write the Maclaurin series for the function $\dfrac{x^2}{(7+x)^2}$.

40. Write the Maclaurin series for $3x^2\cos\left(\dfrac{x^3}{3}\right)$.

41. Write the Maclaurin series for $x^3 e^{-2x^2}$.

42. Calculate $\displaystyle\int \sin\left(3x^4\right)\,dx$ as a Maclaurin series.

43. Find the sum of the following series: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}\pi^{2n}}{6^{2n}(2n)!}$

44. Find the Taylor series for the function $f(x)=\dfrac{2}{x^2}$ centered at $a=-5$.

45. Find the second degree Taylor polynomial, $T_2(x)$, for $f(x)=\cos(2x)$ centered at $a=\dfrac{\pi}{6}$.