MATH 152
SPRING 2008

SAMPLE EXAM I

1. Find the area of the region bounded by the curves $y = x^2$ and $y = 4$.

a) $\frac{32}{3}$ b) $\frac{256}{3}$ c) $\frac{256\pi}{5}$

d) $\frac{32\pi}{5}$ e) $\frac{64}{3}$

2. Which of the following integrals gives the area of the region bounded by $x+y^2 = 0$ and $x - 3y = -4$?

a) $\int_{-4}^{1}(-y^2 - 3y + 4)\,dy$ b) $\int_{-16}^{-1}\left(\frac{1}{3}(x+4) - \sqrt{-x}\right)\,dx$

c) $\int_{-16}^{-1}\left(-\frac{1}{3}(x+4) - \sqrt{-x}\right)\,dx$ d) $\int_{-1}^{4}(-y^2 + 3y + 4)\,dy$

e) no solution-functions not defined

3. Find the volume of the solid formed by rotating the region in the first quadrant bounded by $y = x$ and $y = x^3$ about the x-axis.

a) $\frac{\pi}{4}$ b) $\frac{4\pi}{15}$ c) $\frac{4\pi}{21}$

d) $\frac{2\pi}{21}$ e) $\frac{2\pi}{15}$

4. Find the volume of the solid formed by rotating the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$ about the line $x = 3$.

a) $3\pi e - 4\pi$ b) $\frac{29\pi}{2} + \frac{1}{2}\pi e^2 - 6\pi e$ c) $6\pi e - 8\pi$

d) $\frac{29\pi}{4} + \frac{1}{4}\pi e^2 - 3\pi e$ e) $-2\pi + 2\pi e$

5. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

a) $\frac{75}{4}$ J b) 45 J c) $\frac{75}{2}$ J

d) 30 J e) 40 J

6. Evaluate $\int_0^{\sqrt[3]{\pi/2}} x^5 \cos(x^3)\, dx$

a) $\frac{\pi}{6} - \frac{1}{3}$ b) $\frac{\pi}{3} - \frac{1}{6}$ c) $\frac{\pi}{2} - \frac{1}{3}$

d) $\frac{\pi}{3} - \frac{1}{2}$ e) $\frac{\pi}{6} - \frac{1}{2}$

7. $\int_1^{e^2} \ln x \, dx =$

a) $1 - e^2$ b) $e^2 - 1$ c) $e - 1$

d) $1 + e^2$ e) $1 - e$

8. After making an appropriate substitution, $\int_{\sqrt{3}}^{2} \sqrt{4 - x^2} \, dx =$

a) $4 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$

b) $4 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$

c) $2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \tan \theta \, d\theta$

d) $2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos \theta \, d\theta$

e) $4 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sec \theta \tan^2 \theta \, d\theta$

9. $\int_0^1 \frac{dx}{(x+2)^2(x+4)} =$

a) $\frac{1}{12} + \frac{1}{4}\ln\frac{5}{6}$ b) $\frac{1}{6} + \frac{1}{2}\ln\frac{5}{6}$ c) $3 + 2\ln\frac{6}{5}$

d) $\frac{1}{4} + 2\ln\frac{5}{6}$ e) $\frac{1}{2}\ln\frac{10}{3}$

10. Which of the following is the partial fraction decomposition of $\dfrac{x+3}{(x+1)^2(x^2+2x+3)}$?

a) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x^2+2x+3}$

b) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2+2x+3}$

c) $\dfrac{A}{x+1}+\dfrac{Bx+C}{(x+1)^2}+\dfrac{Dx+E}{x^2+2x+3}$

d) $\dfrac{A}{x+3}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}+\dfrac{Dx+E}{x^2+2x+3}$

e) $\dfrac{A}{x+3}+\dfrac{B}{(x+1)}+\dfrac{C}{(x+1)^2}+\dfrac{D}{x+3}+\dfrac{E}{x-1}$

11. Evaluate $\int_0^{\pi/4} \tan^3 x \sec^4 x \, dx$.

a) 1 b) $\frac{1}{4}$ c) $\frac{5}{12}$ d) $\frac{1}{\pi}$ e) $\frac{2}{3\pi}$

12. An object is moved along the x-axis by a force of magnitude $F(x) = e^{-2x}$. How much work is done as the object moves from $x = 0$ to $x = 3$?

a) $\frac{1}{2}(1 - e^{-6})$ b) $2(e^6 - 1)$ c) $\frac{1}{2}(e^6 + 1)$

d) $2(e^{-6} - 1)$

e) It cannot be determined without knowing the mass or weight of the object.

13. Calculate $\int_0^2 \frac{x+1}{x^2+4}\,dx$.

a) $\frac{\pi}{8} + \ln\sqrt{2}$ b) $\frac{1}{2\pi}$ c) $\ln 128$ d) $\frac{\pi}{6} + \ln\sqrt{3}$

e) $\frac{\pi}{6} + \ln 2$

14. Find the average value of the function $f(x) = \cos^3 x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

a) $\frac{1}{3}$ b) $\frac{\pi}{2}$ c) $\frac{3}{\pi}$ d) $\frac{1}{2}$ e) $\frac{4}{3\pi}$

Part II - Work Out Problems
CALCULATORS ARE NOT ALLOWED ON THIS PART OF THE EXAM

15. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the x-axis.

16. The base of a solid is the triangle with vertices $(0, 0)$, $(0, 4)$, and $(3, 0)$. Cross-sections perpendicular the the x-axis are semicircles. Find the volume of the solid.

17. A tank is in the shape of the region bounded by $y = x^2$, $y = 4$, $x = 0$, revolved around the y axis. Assuming the tank is initially full of water, find the work done in pumping the water to the top of the tank. Assume the tank measurements are in feet and the weight density of water is $\rho g = 62.5$ pounds per cubic foot.

18. Compute $\int \sqrt{x} \ln x \, dx$

19. Compute $\int \frac{x^3}{x^2 + x - 2}\, dx$

20. Compute $\int \frac{dx}{\sqrt{x^2-16}}$

21. Find the area of the region bounded by $y = \cos^2 x$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{6}$