

MATH 152 Sample Exam III
SPRING 2009

Part I - Multiple Choice

1. Find the coefficient of $(x-4)^3$ in the Taylor Series expansion for the function $f(x) = \sqrt{x}$ at $a = 4$.

- a) $\frac{3}{256}$ b) $\frac{3}{128}$ c) $\frac{1}{256}$
d) $\frac{5}{264}$ e) $\frac{1}{512}$

2. Find the limit of the sequence $a_n = \frac{(-1)^n(2n^2 + 2)}{3n^2 + 1}$

- a) $\frac{3}{2}$ b) diverges c) 0
d) 1 e) $\frac{2}{3}$

3. $\sum_{n=1}^{\infty} \frac{3^{n-1} + (-1)^n}{7^n} =$

- a) $\frac{1}{12}$ b) diverges c) $\frac{1}{8}$
d) $\frac{1}{2}$ e) $\frac{21}{8}$

4. $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)} =$

- a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$
d) 0 e) diverges

5. The series $\sum_{n=1}^{\infty} \frac{n}{n^3 + 5}$

- a) Diverges by the Ratio Test
- b) Converges by the Ratio Test
- c) Diverges by the Limit Comparison Test
- d) Converges by the Divergence Test
- e) Converges by the Comparison Test

6. Which of the following series converges?

(I) $\sum_{n=2}^{\infty} \frac{n^3}{n^4 + 5}$ (II) $\sum_{n=1}^{\infty} \frac{n!}{n^{2012}}$ (III) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

- a) I only b) II only c) III only
- d) I and III e) all 3 series converge

7. Which of the following series is absolutely convergent?

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.998}}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.001}}$

c) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$

d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

e) All of these series are absolutely convergent

8. Find the radius of convergence of the power series (i) $\sum_{n=0}^{\infty} \frac{x^n}{(2n+1)!}$ and (ii) $\sum_{n=0}^{\infty} (2n+1)!x^n$

- a) (i) 2 and (ii) 0 b) (i) ∞ and (ii) 0 c) (i) 0 and (ii) ∞
- d) (i) $\frac{1}{2}$ and (ii) 2 e) (i) 1 and (ii) 1

9. Which of the following is a Maclaurin Series for $\int_0^x \sin(t^2) dt$?

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+4)(2n+1)!}$

d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+3)(2n)!}$

10. Find the second-degree Taylor polynomial of $f(x) = e^{-2x}$ centered at $x = -1$.

a) $T_2(x) = e^2 - 2e^2(x+1) + 2e^2(x+1)^2$

b) $T_2(x) = e^2 - 2e^2(x-1) + 2e^2(x-1)^2$

c) $T_2(x) = e^2 - 2e^2(x+1) + 4e^2(x+1)^2$

d) $T_2(x) = e^2 - e^2(x+1) + e^2(x+1)^2$

e) $T_2(x) = e^2 - 2e^2(x-1) + 4e^2(x-1)^2$

11. If the n th partial sum of the series $\sum_{n=0}^{\infty} a_n$ is $s_n = \frac{n+1}{n+2}$, find (i) a_n and (ii) $\sum_{n=0}^{\infty} a_n$.

a) (i) $a_n = \frac{1}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

b) (i) $a_n = \frac{2}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

c) (i) $a_n = \frac{1}{(n+1)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

d) (i) $a_n = \frac{n}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

e) None of these, the series diverges by the Test for Divergence.

12. Which of the following statements is always true? Circle all that apply.

a) The series $\sum_{n=1}^{\infty} ar^n$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

b) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges absolutely if $p \geq 1$.

c) The Ratio Test fails when applied to the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n + 1}$

d) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

e) If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

13. Find the center and radius of the sphere $x^2 + y^2 + z^2 + x + 2y - 6z = 0$.

a) $C(\frac{1}{2}, 1, -3), r = \frac{\sqrt{41}}{2}$

b) $C(-\frac{1}{2}, -1, 3), r = \frac{\sqrt{41}}{2}$

c) $C(-\frac{1}{2}, -1, 3), r = \frac{41}{2}$

d) $C(-\frac{1}{2}, 1, -3), r = \frac{41}{2}$

e) $C(-\frac{1}{2}, 1, -3), r = \frac{\sqrt{41}}{2}$

Part II - Work Out Problems

14. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$ converges or diverges.

15. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ converges or diverges.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n(2^n)}$

17. Using a power series about $a = 0$ for $f(x) = \frac{1}{1+x^2}$, find a power series about $a = 0$ for $g(x) = \ln(1+x^2)$.

18. Find the Taylor Series for $f(x) = \ln x$ centered at $a = 1$.

19. Approximate $f(x) = \sqrt{x}$ with a second degree Taylor Polynomial at $x = 4$. Use Taylor's Inequality to estimate the accuracy of the approximation $\sqrt{x} \approx T_2(x)$ for $3 \leq x \leq 4.2$.

20. Test the series $\sum_{n=1}^{\infty} \frac{1}{n+n \cos^2 n}$ for convergence.

21. Approximate $\int_0^1 e^{-x^2} dx$ with error less than $\frac{1}{100}$.

22. Prove the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$ converges. Use the sum of the first 5 terms to approximate the sum of the series and estimate the error.