

**MATH 152 Sample Exam III**  
**SPRING 2008**

**Part I - Multiple Choice**

1. Find the coefficient of  $(x-4)^3$  in the Taylor Series expansion for the function  $f(x) = \sqrt{x}$  at  $a = 4$ .

- a)  $\frac{3}{256}$                       b)  $\frac{3}{128}$                       c)  $\frac{1}{256}$   
d)  $\frac{5}{264}$                       e)  $\frac{1}{512}$

2. Find the limit of the sequence  $a_n = \frac{(-1)^n(2n^2 + 2)}{3n^2 + 1}$

- a)  $\frac{3}{2}$                       b) diverges                      c) 0  
d) 1                      e)  $\frac{2}{3}$

3.  $\sum_{n=1}^{\infty} \frac{3^{n-1}}{5^n} =$

- a)  $\frac{14}{29}$                       b) diverges                      c)  $\frac{5}{6}$   
d)  $\frac{1}{2}$                       e)  $\frac{5}{7}$

4.  $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)} =$

- a)  $\frac{1}{2}$                       b)  $\frac{1}{3}$                       c)  $\frac{1}{4}$   
d) 0                      e) diverges

5. The series  $\sum_{n=1}^{\infty} \frac{n}{n^3 + 5}$

- a) Diverges by the Ratio Test
- b) Converges by the Ratio Test
- c) Diverges by the Limit Comparison Test
- d) Converges by the Divergence Test
- e) Converges by the Comparison Test

6. Which of the following series converges?

(I)  $\sum_{n=2}^{\infty} \frac{n^3}{n^4 + 5}$

(II)  $\sum_{n=1}^{\infty} \frac{n!}{n^{2006}}$

(III)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

- a) I only
- b) II only
- c) III only
- d) I and III
- e) all 3 series converge

7. Which of the following series is absolutely convergent?

a)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.998}}$

b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.001}}$

c)  $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$

d)  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

e) All of these series are absolutely convergent

8. Find the radius of convergence of the power series  $\sum_{n=0}^{\infty} \frac{x^n}{(2n+1)!}$

- a) 2
- b)  $\infty$
- c) 0
- d)  $\frac{1}{2}$
- e) 1

9. Which of the following is a Maclaurin Series for  $\int_0^x \sin(t^2) dt$ ?

a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$

b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$

c)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+4)(2n+1)!}$

d)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

e)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+3)(2n)!}$

10. Find the second-degree Taylor polynomial of  $f(x) = e^{-2x}$  centered at  $x = -1$ .

a)  $T_2(x) = e^2 - 2e^2(x+1) + 2e^2(x+1)^2$

b)  $T_2(x) = e^2 - 2e^2(x-1) + 2e^2(x-1)^2$

c)  $T_2(x) = e^2 - 2e^2(x+1) + 4e^2(x+1)^2$

d)  $T_2(x) = e^2 - e^2(x+1) + e^2(x+1)^2$

e)  $T_2(x) = e^2 - 2e^2(x-1) + 4e^2(x-1)^2$

11. Find a unit vector in the direction of  $\mathbf{a} = \langle 1, -2, 4 \rangle$ .

a)  $\left\langle \frac{1}{\sqrt{21}}, \frac{-2}{\sqrt{21}}, \frac{4}{\sqrt{21}} \right\rangle$

b)  $\left\langle \frac{1}{\sqrt{7}}, \frac{-2}{\sqrt{7}}, \frac{4}{\sqrt{7}} \right\rangle$

c)  $\left\langle -\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}} \right\rangle$

d)  $\left\langle -\frac{1}{\sqrt{7}}, \frac{2}{\sqrt{7}}, -\frac{4}{\sqrt{7}} \right\rangle$

e) None of these.

12. Which of the following statements is always true? Circle all that apply.

a) The series  $\sum_{n=1}^{\infty} ar^n$  converges if  $|r| < 1$  and diverges if  $|r| \geq 1$ .

b) The series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges absolutely if  $p \geq 1$ .

c) The Ratio Test fails when applied to the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n + 1}$

d) If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum_{n=1}^{\infty} a_n$  converges.

e) If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then  $\sum_{n=1}^{\infty} a_n$  diverges.

13. Find the center and radius of the sphere  $x^2 + y^2 + z^2 + x + 2y - 6z = 0$ .

a)  $C\left(\frac{1}{2}, 1, -3\right), r = \frac{\sqrt{41}}{2}$

b)  $C\left(-\frac{1}{2}, -1, 3\right), r = \frac{\sqrt{41}}{2}$

c)  $C\left(-\frac{1}{2}, -1, 3\right), r = \frac{41}{2}$

d)  $C\left(-\frac{1}{2}, 1, -3\right), r = \frac{41}{2}$

e)  $C\left(-\frac{1}{2}, 1, -3\right), r = \frac{\sqrt{41}}{2}$

## Part II - Work Out Problems

14. Determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$  converges or diverges.

15. Determine whether the series  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$  converges or diverges.

16. Find the radius and interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n(2^n)}$

17. Using a geometric series about  $a = 0$  for  $f(x) = \frac{1}{1+x^2}$ , find a power series about  $a = 0$  for

$$g(x) = \frac{2x}{(1+x^2)^2}.$$

18. Find the Taylor Series for  $f(x) = \ln x$  centered at  $a = 1$ .

19. Find the cosine of the angle between the vectors  $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{b} = 5\mathbf{j} + \mathbf{k}$ .

20. Given  $\mathbf{a} = \langle 1, 3, 2 \rangle$ ,  $\mathbf{b} = \langle 1, -4, 1 \rangle$ , find the scalar and vector projection of  $\mathbf{b}$  onto  $\mathbf{a}$ .

21. Approximate  $\int_0^1 e^{-x^2} dx$  with error less than  $\frac{1}{100}$ .

22. Prove the series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$  converges. Use the sum of the first 5 terms to approximate the sum of the series and estimate the error.