

MATH 152 Sample Exam III
SPRING 2008

Part I - Multiple Choice

1. Find the coefficient of $(x-4)^3$ in the Taylor Series expansion for the function $f(x) = \sqrt{x}$ at $a = 4$.

a) $\frac{3}{256}$

b) $\frac{3}{128}$

c) $\frac{1}{256}$

d) $\frac{5}{264}$

e) $\frac{1}{512}$

2. Find the limit of the sequence $a_n = \frac{(-1)^n(2n^2 + 2)}{3n^2 + 1}$

a) $\frac{3}{2}$

b) diverges

c) 0

d) 1

e) $\frac{2}{3}$

$$3. \sum_{n=1}^{\infty} \frac{3^{n-1}}{5^n} =$$

a) $\frac{14}{29}$

b) diverges

c) $\frac{5}{6}$

d) $\frac{1}{2}$

e) $\frac{5}{7}$

$$4. \sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)} =$$

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) 0

e) diverges

5. The series $\sum_{n=1}^{\infty} \frac{n}{n^3 + 5}$

- a) Diverges by the Ratio Test
- b) Converges by the Ratio Test
- c) Diverges by the Limit Comparison Test
- d) Converges by the Divergence Test
- e) Converges by the Comparison Test

6. Which of the following series converges?

$$(I) \sum_{n=2}^{\infty} \frac{n^3}{n^4 + 5}$$

$$(II) \sum_{n=1}^{\infty} \frac{n!}{n^{2006}}$$

$$(III) \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$

a) I only

b) II only

c) III only

d) I and III

e) all 3 series converge

7. Which of the following series is absolutely convergent?

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.998}}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.001}}$

c) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$

d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

e) All of these series are absolutely convergent

8. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{(2n+1)!}$

a) 2

b) ∞

c) 0

d) $\frac{1}{2}$

e) 1

9. Which of the following is a Maclaurin Series for $\int_0^x \sin(t^2) dt$?

a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+4)(2n+1)!}$

d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+3)(2n)!}$

10. Find the second-degree Taylor polynomial of $f(x) = e^{-2x}$ centered at $x = -1$.

a) $T_2(x) = e^2 - 2e^2(x + 1) + 2e^2(x + 1)^2$

b) $T_2(x) = e^2 - 2e^2(x - 1) + 2e^2(x - 1)^2$

c) $T_2(x) = e^2 - 2e^2(x + 1) + 4e^2(x + 1)^2$

d) $T_2(x) = e^2 - e^2(x + 1) + e^2(x + 1)^2$

e) $T_2(x) = e^2 - 2e^2(x - 1) + 4e^2(x - 1)^2$

11. Find a unit vector in the direction of $\mathbf{a} = \langle 1, -2, 4 \rangle$.

a) $\left\langle \frac{1}{\sqrt{21}}, \frac{-2}{\sqrt{21}}, \frac{4}{\sqrt{21}} \right\rangle$

b) $\left\langle \frac{1}{\sqrt{7}}, \frac{-2}{\sqrt{7}}, \frac{4}{\sqrt{7}} \right\rangle$

c) $\left\langle -\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}} \right\rangle$

d) $\left\langle -\frac{1}{\sqrt{7}}, \frac{2}{\sqrt{7}}, -\frac{4}{\sqrt{7}} \right\rangle$

e) None of these.

12. Which of the following statements is always true? Circle all that apply.

a) The series $\sum_{n=1}^{\infty} ar^n$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

b) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges absolutely if $p \geq 1$.

c) The Ratio Test fails when applied to the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n + 1}$

d) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

e) If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

13. Find the center and radius of the sphere $x^2 + y^2 + z^2 + x + 2y - 6z = 0$.

a) $C(\frac{1}{2}, 1, -3), r = \frac{\sqrt{41}}{2}$

b) $C(-\frac{1}{2}, -1, 3), r = \frac{\sqrt{41}}{2}$

c) $C(-\frac{1}{2}, -1, 3), r = \frac{41}{2}$

d) $C(-\frac{1}{2}, 1, -3), r = \frac{41}{2}$

e) $C(-\frac{1}{2}, 1, -3), r = \frac{\sqrt{41}}{2}$

Part II - Work Out Problems

14. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 1}$ converges or diverges.

15. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ converges or diverges.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n(2^n)}$

17. Using a geometric series about $a = 0$ for $f(x) = \frac{1}{1+x^2}$, find a power series about $a = 0$ for $g(x) = \frac{2x}{(1+x^2)^2}$.

18. Find the Taylor Series for $f(x) = \ln x$ centered at $a = 1$.

19. Find the cosine of the angle between the vectors $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 5\mathbf{j} + \mathbf{k}$.

20. Given $\mathbf{a} = \langle 1, 3, 2 \rangle$, $\mathbf{b} = \langle 1, -4, 1 \rangle$, find the scalar and vector projection of \mathbf{b} onto \mathbf{a} .

21. Approximate $\int_0^1 e^{-x^2} dx$ with error less than $\frac{1}{100}$.

22. Prove the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$ converges. Use the sum of the first 5 terms to approximate the sum of the series and estimate the error.