

**Spring 2007 Math 152 Practice Final**  
**Answers to Amy Austin's Problems**  
**Tue, 01/May ©2007, Art Belmonte**

1. (a)  $\int \frac{x-2}{x(x^2+1)} dx = \ln\left(\frac{x^2+1}{x^2}\right) + \tan^{-1}x + C$   
 (b)  $\int_{\sqrt{2}}^2 \frac{1}{\sqrt{x^2-1}} dx = \ln\left(\left(\sqrt{3}+2\right)\left(\sqrt{2}-1\right)\right) \approx 0.436$   
 (c)  $\int \cos^4(2x) dx = \frac{1}{64} \sin 8x + \frac{1}{8} \sin 4x + \frac{3}{8}x + C$   
 (d)  $\int x \sin 2x dx = \frac{1}{4} \sin 2x - \frac{1}{2}x \cos 2x + C$
2.  $\int_0^{\infty} \frac{2}{2x+1} - \frac{1}{x+3} dx = \ln 6 \approx 1.79$
3.  $A = \int_0^{\pi/2} \sin^3 x dx = \frac{2}{3} \approx 0.67 \text{ cm}^2$
4. (a)  $V = \int_{-1}^1 \pi(4-x^2)^2 - \pi(3)^2 dx = \frac{136\pi}{15} \approx 28.48 \text{ cm}^3$   
 (b)  $V = \int_3^4 2\pi y \cdot 2\sqrt{4-y} dy = \frac{136\pi}{15} \approx 28.48 \text{ cm}^3$
5.  $V = \int_0^3 \frac{1}{2}\pi\left(\frac{1}{2}\left(4-\frac{4}{3}x\right)\right)^2 dx = 2\pi \approx 6.28 \text{ cm}^3$
6.  $y^3 = C - \frac{3 \ln x}{2x^2} - \frac{3}{4x^2}$  or  $y = \sqrt[3]{C - \frac{3 \ln x}{2x^2} - \frac{3}{4x^2}}$
7.  $f_{\text{ave}} = \frac{1}{\frac{\pi}{4}-0} \int_0^{\pi/4} \tan x dx = \frac{2 \ln 2}{\pi} \approx 0.441$
8.  $y = 250 - 150e^{-t/100}$
9. (a)  $S = \int_0^1 2\pi y^{3/2} \sqrt{1 + \left(\frac{3}{2}y^{1/2}\right)^2} dy$   
 $= \frac{4\pi(2 \ln(3 + \sqrt{13}) - 2 \ln 2 + 21\sqrt{13})}{243} \approx 4.04 \text{ cm}^2$   
 (b)  $S = \int_0^1 2\pi y \sqrt{1 + \left(\frac{3}{2}y^{1/2}\right)^2} dy$   
 $= \frac{2\pi(64 + 247\sqrt{13})}{1215} \approx 4.94 \text{ cm}^2$
10. (a)  $W = \int_{-3}^0 62.5 \cdot 2\sqrt{3^2 - y^2} \cdot 8(0-y) dy = 9000 \text{ ft-lb}$   
 (b)  $F = \int_{-3}^0 62.5(0-y) \cdot 2\sqrt{3^2 - y^2} dy = 1125 \text{ lb}$
11. The work done in pulling half the rope to the top is  
 $W = \int_0^{50} 2x dx + (50)(2)(50) = 7500 \text{ foot pounds.}$
12. mass:  $m = \int_0^1 \int_0^{e^{4x}} k dy dx = \frac{1}{4}k(e^4 - 1)$   
 $[\bar{x}, \bar{y}] = \frac{1}{m} \int_0^1 \int_0^{e^{4x}} k[x, y] dy dx = \left[ \frac{3e^4 + 1}{4(e^4 - 1)}, \frac{e^4 + 1}{4} \right]$   
 $\approx [0.77, 13.90]$
13.  $\ln(1+2x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n}{n} x^n$

14.  $W = \int_0^3 100x dx = 450 \text{ ft-lb}$
15. (a) Ignore the first term, which has no bearing on the convergence of the infinite series. We then have  
 $\sum a_n = \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5+10}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = \sum b_n,$   
 a divergent  $p$ -series ( $p = \frac{1}{2} \leq 1$ ). Hence  $\sum a_n$  diverges by the Limit Comparison Test since  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1$ .
- (b) The Ratio Test yields  
 $\left| \frac{a_{n+1}}{a_n} \right| = \frac{((n+1)!)^2 \cdot (2n)!}{(2(n+1))! \cdot (n!)^2} = \frac{(n+1)^2}{(2n+1)(2n+2)} \rightarrow \frac{1}{4} < 1.$   
 Thus the series  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$  converges (absolutely).
- (c) Since  $b_n = |a_n| = \frac{1}{\sqrt{n+7}} \downarrow 0$ , we conclude by the Alternating Series Test that the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$  converges.
- (d) Since  $\int_0^{\infty} \frac{\ln(x+1)}{x+1} dx = \infty$ , the Integral Test tells us that the series  $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1}$  diverges.
16. We have  $T_2(x) = e^{-1} - e^{-1}(x-1) + \frac{1}{2}e^{-1}(x-1)^2$ .  
 With  $f(x) = e^{-x}$ , we have  $|f^{(n)}(x)| = e^{-x}$ . Accordingly, Taylor's Inequality gives  

$$|R_2(x)| \leq \frac{\max_{0.5 \leq x \leq 1.1} |f^{(3)}(x)|}{3!} \left( \max_{0.5 \leq x \leq 1.1} |x-1| \right)^3$$

$$= \frac{e^{-1/2} \left(\frac{1}{2}\right)^3}{6} = \frac{e^{-1/2}}{48} \approx 1.264 \times 10^{-2}.$$
17. An equation of the sphere is  
 $(x-2)^2 + (y+4)^2 + (z-1)^2 = 89.$
18. A unit vector perpendicular to the triangle with the stated vertices is given by  $\hat{\mathbf{v}} = \mathbf{v} / \|\mathbf{v}\|$ , where  
 $\mathbf{v} = \overrightarrow{PQ} \times \overrightarrow{PR} = (\overrightarrow{Q} - \overrightarrow{P}) \times (\overrightarrow{R} - \overrightarrow{P}) = [9, -6, 15]$ . Thus  
 $\hat{\mathbf{v}} = \left[ \frac{9}{\sqrt{342}}, -\frac{6}{\sqrt{342}}, \frac{15}{\sqrt{342}} \right] = \left[ \frac{3\sqrt{38}}{38}, -\frac{\sqrt{38}}{19}, \frac{5\sqrt{38}}{38} \right]$   
 $\approx [0.49, -0.32, 0.81].$