

Spring 2007 Math 152 Practice Final
Answers to Amy Austin's Problems
Tue, 01/May ©2007, Art Belmonte

- $\int \frac{x-2}{x(x^2+1)} dx = \ln\left(\frac{x^2+1}{x^2}\right) + \tan^{-1}x + C$
 - $\int_{\sqrt{2}}^2 \frac{1}{\sqrt{x^2-1}} dx = \ln\left(\left(\sqrt{3}+2\right)\left(\sqrt{2}-1\right)\right) \approx 0.436$
 - $\int \cos^4(2x) dx = \frac{1}{64} \sin 8x + \frac{1}{8} \sin 4x + \frac{3}{8}x + C$
 - $\int x \sin 2x dx = \frac{1}{4} \sin 2x - \frac{1}{2}x \cos 2x + C$
- $\int_0^{\infty} \frac{2}{2x+1} - \frac{1}{x+3} dx = \ln 6 \approx 1.79$
- $A = \int_0^{\pi/2} \sin^3 x dx = \frac{2}{3} \approx 0.67 \text{ cm}^2$
- $V = \int_{-1}^1 \pi(4-x^2)^2 - \pi(3)^2 dx = \frac{136\pi}{15} \approx 28.48 \text{ cm}^3$
 - $V = \int_3^4 2\pi y \cdot 2\sqrt{4-y} dy = \frac{136\pi}{15} \approx 28.48 \text{ cm}^3$
- $V = \int_0^3 \frac{1}{2} \pi \left(\frac{1}{2}(4-\frac{4}{3}x)\right)^2 dx = 2\pi \approx 6.28 \text{ cm}^3$
- $y^3 = C - \frac{3 \ln x}{2x^2} - \frac{3}{4x^2}$ or $y = \sqrt[3]{C - \frac{3 \ln x}{2x^2} - \frac{3}{4x^2}}$
- $f_{\text{ave}} = \frac{1}{\frac{\pi}{4}-0} \int_0^{\pi/4} \tan x dx = \frac{2 \ln 2}{\pi} \approx 0.441$
- $y = 250 - 150e^{-t/100}$
- $S = \int_0^1 2\pi y^{3/2} \sqrt{1 + \left(\frac{3}{2}y^{1/2}\right)^2} dy$
 $= \frac{4\pi(2 \ln(3 + \sqrt{13}) - 2 \ln 2 + 21\sqrt{13})}{243} \approx 4.04 \text{ cm}^2$
 - $S = \int_0^1 2\pi y \sqrt{1 + \left(\frac{3}{2}y^{1/2}\right)^2} dy$
 $= \frac{2\pi(64 + 247\sqrt{13})}{1215} \approx 4.94 \text{ cm}^2$
- $W = \int_{-3}^0 62.5 \cdot 2\sqrt{3^2-y^2} \cdot 8(0-y) dy = 9000 \text{ ft-lb}$
 - $F = \int_{-3}^0 62.5(0-y) \cdot 2\sqrt{3^2-y^2} dy = 1125 \text{ lb}$
- The work done in pulling half the rope to the top is
 $W = \int_0^{50} 2x dx + (50)(2)(50) = 7500 \text{ foot pounds.}$
- mass: $m = \int_0^1 \int_0^{e^{4x}} k dy dx = \frac{1}{4}k(e^4 - 1)$
 $[\bar{x}, \bar{y}] = \frac{1}{m} \int_0^1 \int_0^{e^{4x}} k[x, y] dy dx = \left[\frac{3e^4 + 1}{4(e^4 - 1)}, \frac{e^4 + 1}{4} \right]$
 $\approx [0.77, 13.90]$
- $\ln(1+2x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n}{n} x^n$

- $W = \int_0^3 100x dx = 450 \text{ ft-lb}$
- Ignore the first term, which has no bearing on the convergence of the infinite series. We then have
 $\sum a_n = \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5+10}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = \sum b_n$,
a divergent p -series ($p = \frac{1}{2} \leq 1$). Hence $\sum a_n$ diverges by the Limit Comparison Test since $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1$.
 - The Ratio Test yields
 $\left| \frac{a_{n+1}}{a_n} \right| = \frac{((n+1)!)^2 \cdot (2n)!}{(2(n+1))! \cdot (n!)^2} = \frac{(n+1)^2}{(2n+1)(2n+2)} \rightarrow \frac{1}{4} < 1$.
Thus the series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ converges (absolutely).
 - Since $b_n = |a_n| = \frac{1}{\sqrt{n+7}} \downarrow 0$, we conclude by the Alternating Series Test that the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$ converges.
 - Since $\int_0^{\infty} \frac{\ln(x+1)}{x+1} dx = \infty$, the Integral Test tells us that the series $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1}$ diverges.
- We have $T_2(x) = e^{-1} - e^{-1}(x-1) + \frac{1}{2}e^{-1}(x-1)^2$.
With $f(x) = e^{-x}$, we have $|f^{(n)}(x)| = e^{-x}$. Accordingly, Taylor's Inequality gives

$$|R_2(x)| \leq \frac{\max_{0.5 \leq x \leq 1.1} |f^{(3)}(x)|}{3!} \left(\max_{0.5 \leq x \leq 1.1} |x-1| \right)^3$$

$$= \frac{e^{-1/2}}{6} \left(\frac{1}{2} \right)^3 = \frac{e^{-1/2}}{48} \approx 1.264 \times 10^{-2}.$$
- An equation of the sphere is
 $(x-2)^2 + (y+4)^2 + (z-1)^2 = 89.$
- A unit vector perpendicular to the triangle with the stated vertices is given by $\hat{\mathbf{v}} = \mathbf{v} / \|\mathbf{v}\|$, where
 $\mathbf{v} = \overrightarrow{PQ} \times \overrightarrow{PR} = (\overrightarrow{Q} - \overrightarrow{P}) \times (\overrightarrow{R} - \overrightarrow{P}) = [9, -6, 15]$. Thus

$$\hat{\mathbf{v}} = \left[\frac{9}{\sqrt{342}}, -\frac{6}{\sqrt{342}}, \frac{15}{\sqrt{342}} \right] = \left[\frac{3\sqrt{38}}{38}, -\frac{\sqrt{38}}{19}, \frac{5\sqrt{38}}{38} \right]$$
 $\approx [0.49, -0.32, 0.81].$