Spring 2013 Math 152

courtesy: Amy Austin
(covering sections 8.3-10.2)

Section 8.3

1. $\int \frac{dx}{x^2\sqrt{x^2-1}}=$

2. $\int_0^2 x^3\sqrt{x^2+4}\,dx=$

3. $\int \sqrt{-x^2+6x+7}\,dx=$

Section 8.4

4. $\int_2^3 \frac{x^3+1}{x^2(x-1)}\,dx=$

5. $\int \frac{x+1}{x^2-4}\,dx=$

6. $\int \frac{2x^2-x+4}{x^3+4x}\,dx=$

Section 8.9

7. $\int_e^\infty \frac{dx}{x(\ln x)^2}=$

8. $\int_1^9 \frac{1}{\sqrt[3]{x-9}}\,dx=$

9. $\int_{-1}^2 \frac{1}{x^4}\,dx=$

10. Use the comparison theorem to determine whether the following improper integrals converge or diverge:

    a.) $\int_1^\infty \frac{1}{x+e^{2x}}\,dx$

    b.) $\int_5^\infty \frac{x}{x^{3/2}-x-1}\,dx$

Section 9.3

11. Find the length of the curve $x=\frac{4\sqrt{2}}{3}y^{3/2}-1$ from $y=0$ to $y=1$.

12. Find the length of the curve $y=\frac{x^3}{6}+\frac{1}{2x}$ from the point $(1,2/3)$ to the point $(3,14/3)$.

13. Find the length of the curve $x=\cos t$, $y=\sin t$, $0\le t\le\frac{\pi}{3}$.

Section 9.4

14. Find the surface area of the region obtained by rotating the curve $y=x^2$, $0\le x\le 2$, about the y axis.

15. Find the surface area of the region obtained by rotating the curve $y=\sqrt{x}$, $1\le x\le 4$ about the x axis.

16. Find the surface area obtained by rotating the curve $x=t^3$, $y=t^2$, $0\le t\le 1$, about the x-axis.

Section 10.1

17. Discuss the convergence or divergence of the following sequences:

    a.) $a_n=\ln(3n+1)-\ln(4n^2)$

    b.) $a_n=(-1)^n\frac{n}{n+1}$

    c.) $a_n=(-1)^n\frac{n}{n^2+1}$

    d.) $a_n=\sqrt{n^2-8n}-n$

18. Determine whether the sequence is bounded:

    a.) $a_n=\left\{\frac{1}{n^2}\right\}_{n=1}^\infty$

    b.) $a_n=\left\{\frac{n^2}{n+1}\right\}_{n=1}^\infty$

19. Determine whether following sequences are increasing, decreasing, or not monotonic.

    a.) $a_n=\frac{3}{n+5}$

    b.) $a_n=\cos\frac{n\pi}{2}$

20. Consider the recursive sequence defined by $a_1 = 2$, $a_{n+1} = 1 - \frac{1}{a_n}$. Find the first 5 terms of the sequence. Find the limit of the sequence, if it exists.

21. Given the recursive sequence below is increasing and bounded, find the limit.

    $a_1 = 2$, $a_{n+1} = 4 - \frac{3}{a_n}$.

Section 10.2

22. Use the Test For Divergence to show the series diverges:

    $\sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)}$

23. Explain why the Test for Divergence is inconclusive when applied to the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.

24. If the n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n+1}{n+4}$, find:

    a.) s_{100}, that is $\sum_{n=1}^{100} a_n =?$

    b.) The sum of the series, that is $\sum_{n=1}^{\infty} a_n =?$

    c.) A general formula for a_n, then find a_6.

25. Find the sum of the series:

    a.) $\sum_{n=1}^{\infty} \left(\sin\frac{1}{n} - \sin\frac{1}{n+1}\right)$

    b.) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

    c.) $\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1}$

    d.) $\sum_{n=1}^{\infty} \frac{3^{2n+1}}{10^n}$

    e.) $\sum_{n=0}^{\infty} \frac{2^{3n}}{(-5)^{n+1}}$

    f.) $\sum_{n=0}^{\infty} \frac{(-1)^n + 3^n}{4^n}$