Spring 2013 Math 152

courtesy: Amy Austin
(covering sections 10.3-11.2)

Section 10.3

1. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

   a.) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

   b.) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n+1}}$

   c.) $\sum_{n=2}^{\infty} \frac{2+\cos n}{n^3+n^2+1}$

   d.) $\sum_{n=2}^{\infty} \frac{n}{n^2+n+1}$

2. Consider the series $\sum_{n=1}^{\infty} ne^{-n^2}$. Prove the series converges using the Integral Test. Use S_6 to estimate the sum of the series and estimate the remainder (error).

Section 10.4

3. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

   a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   b.) $\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{\sqrt{n+1}}$

4. Determine whether the following series diverge, coonverge absolutely, or converge conditionally (converges but not absolutely).

   a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{3n-1}$

   b.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3+1}$

   c.) $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{1}{n}\right)}{n^2}$

   d.) $\sum_{n=1}^{\infty} \frac{(-10)^n n!}{(2n+1)!}$

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$.

   a.) Use the first 5 terms to estimate the sum.

   b.) Estimate the error in the approximation s_5 to the sum of the series.

Section 10.5

6. For the following power series, find the radius and interval of convergence.

   a.) $\sum_{n=1}^{\infty} \frac{(-1)^n(x+1)^n}{2^n\sqrt{n}}$

   b.) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   c.) $\sum_{n=1}^{\infty} \frac{(2n-1)!(x+2)^{n-1}}{5^{n-1}}$

7. If the series $\sum_{n=1}^{\infty} c_n(x-1)^n$ has a radius of convergence of 5, then what do we know about the following series:
   $\sum_{n=1}^{\infty} c_n(3)^n$, $\sum_{n=1}^{\infty} c_n(5.5)^n$

Section 10.6 and 10.7

7. Find a Maclaurin series for the following functions and the associated radius of convergence.

   a.) $f(x) = \frac{1}{8+3x^2}$

   b.) $f(x) = \frac{x}{(1-x^2)^2}$

   c.) $f(x) = \ln(2-x)$

   d.) $f(x) = x^5e^{8x^2}$

   e.) $f(x) = \sin\left(\frac{x}{3}\right)$

   f.) $\int x\arctan(x^3)\,dx$

8. Consider the Taylor Series for $f(x) = \ln x$ centered at 2. What is the coefficient of $(x-2)^4$?

9. Find the Taylor Series for $f(x) = (x+2)e^x$ at $x = 1$.

10. Using the known Maclaurin for $\cos(x)$, find the 40^{th} derivative at $x = 0$ for $f(x) = \cos\left(\frac{x^2}{2}\right)$.

11. Express $\int_0^1 e^{-x^2}\,dx$ as an infinite series. Use the first 2 terms of this series to approximate the sum.

Section 10.9

12. If $f(x) = \sqrt{1+x}$, $n = 2$, $a = 3$, $2 \leq x \leq 3.1$

    a.) Find $T_n(x)$ at the given value of a.

    b.) Use Taylor's Inequality to estimate the accuracy of the approximation $T_n(x)$ for x in the given interval.

Section 11.1 and 11.2

13. Given the points $A(5,5,1)$, $B(3,3,2)$ and $C(1,4,4)$, determine whether triangle ABC is isosceles, right, both, or neither. Also, find the angle located at A.

14. Find the center and radius of the sphere

    $x^2+y^2+z^2+x+2y-2=0$. What is the intersection of this sphere with the xz plane?

15. Given $\mathbf{a} = \langle 1,5,7 \rangle$, $\mathbf{b} = \langle 2,0,5 \rangle$, find

    (i) $2\mathbf{a} - \frac{1}{2}\mathbf{b}$

    (ii) A unit vector in the direction of $\mathbf{b}$

    (iii) The cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

16. Find the value(s) of x so that the vectors

    $\langle x, x, -1 \rangle$ and $\langle 1, x, 6 \rangle$ are orthogonal.

17. Given $\mathbf{c} = \langle 1,3,2 \rangle$, $\mathbf{d} = \langle 1,-4,1 \rangle$, find the scalar and vector projection of $\mathbf{c}$ onto $\mathbf{d}$.