Summer 2012 Math 152

Week in Review 8
courtesy: Amy Austin
(covering section 10.7-10.9)

Section 10.7

1. Find the Taylor Series for f centered at 5 if $f^{(n)}(5) = \dfrac{(-2)^n n!}{7^n(n+5)}$. What is the radius of convergence of this Taylor Series?

2. Find the Taylor Series for $f(x) = e^{2x}$ centered at -1. What is the associated radius of convergence?

3. Find the Taylor Series for $f(x) = \dfrac{1}{x+1}$ at $x = 3$ and the associated radius of convergence.

4. Use a known MacLaurin series derived in this section to obtain a Maclaurin Series for:

   a.) $f(x) = \cos(x^3)$

   b.) $f(x) = xe^{-x}$

   c.) $f(x) = x^5 \sin(\frac{x}{2})$

5. Express $\displaystyle\int \frac{\sin 2x}{x}\, dx$ as an infinite series.

6. Use series to approximate

   $\displaystyle\int_0^{0.5} \cos(x^2)\, dx$ with error less than 10^{-3}

7. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{3n}}{n!}$.

8. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n+1}(2n+1)!}$

9. Find the sum of the series $5 + \dfrac{25}{2} + \dfrac{125}{3!} + \dfrac{625}{4!} + ...$

Section 10.9

10. Find the third degree Taylor Polynomial for

    $f(x) = \sqrt{x}$ at $x = 1$.

11. Find the second degree Taylor Polynomial for $f(x) = \ln x$ at $x = 2$. Using Taylor's Inequality, find an upper bound on the remainder in using $T_2(x)$ to approximate $f(x) = \ln x$ for $1 \leq x \leq 3.2$.

12. Recall $\sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ is the Maclaurin Series for $\sin x$.

    (a) Using the Alternating Series Estimation Theorem, what is the maximum error possible in using the approximation

    $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$ for $|x| < 0.3$?

    (b) Use $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$ to approximate $\sin(1.2°)$.

    (c) Use the Alternating Series Estimation Theorem to estimate the range of values of x for which the given approximation is accurate to within the stated error.

    $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$, error$< 0.000001$.