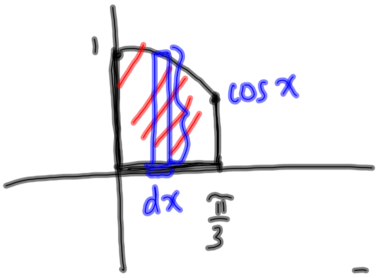
1. Find the area bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \dfrac{\pi}{3}$.



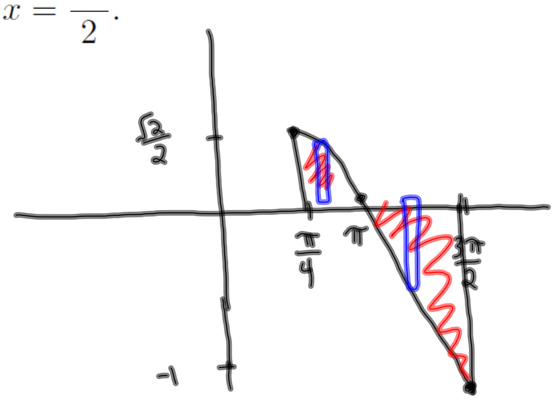

$$A = \int_0^{\frac{\pi}{3}} \cos x \, dx$$

$$= \sin x \Big|_0^{\frac{\pi}{3}}$$

$$= \sin \frac{\pi}{3} - \sin 0$$

$$= \frac{\sqrt{3}}{2} - 0 = \boxed{\frac{\sqrt{3}}{2}}$$

2. Find the area bounded by $y = \sin x$, ~~$y = 0$~~, $x = \dfrac{\pi}{4}$, $x = \dfrac{3\pi}{2}$.




$$A = \int_{\frac{\pi}{4}}^{\pi} \sin x \, dx + \int_{\pi}^{\frac{3\pi}{2}} -\sin x \, dx$$

$$-\cos x \Big|_{\frac{\pi}{4}}^{\pi} + \cos x \Big|_{\pi}^{\frac{3\pi}{2}}$$

$$-(-1) - \left(-\frac{\sqrt{2}}{2}\right) + 0 - (-1)$$

$$1 + \frac{\sqrt{2}}{2} + 1 = \boxed{2 + \frac{\sqrt{2}}{2}}$$

3. Find the area bounded by $y = x^2$ and $y = 2x - x^2$.

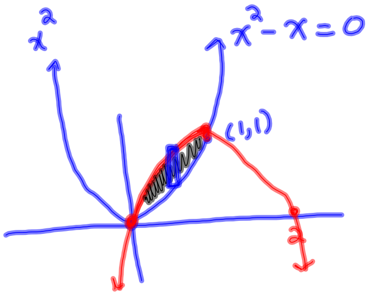
intersection: $x^2 = 2x - x^2$

$2x^2 = 2x$

$x^2 - x = 0 \Rightarrow x(x-1) = 0$

$x = 0,\ y = 0$

$x = 1,\ y = 1$




$2x - x^2$

$x(2-x)$

x-int: $x = 0$

$x = 2$

$$A = \int_0^1 \left(2x - x^2 - x^2\right) dx$$

$$= \int_0^1 \left(2x - 2x^2\right) dx$$

$$= \left(x^2 - \frac{2}{3}x^3\right)\Big|_0^1 = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

4. Find the area bounded by $y = x - 1$ and $y^2 = 2x + 6$.

$x = y + 1,\quad x = \frac{y^2 - 6}{2} \rightarrow x = \frac{1}{2}y^2 - 3$

intersection: $y + 1 = \frac{y^2 - 6}{2}$

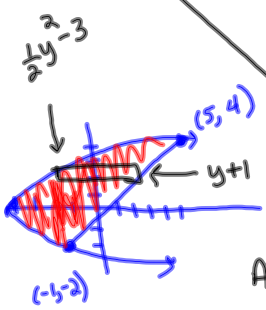
$2y + 2 = y^2 - 6$

$0 = y^2 - 2y - 8$

$0 = (y-4)(y+2)$

$y = 4,\ x = 5$

$y = -2,\ x = -1$

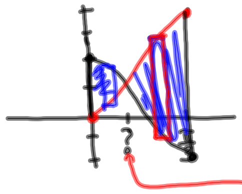



$$A = \int_{-2}^{4} \left[y + 1 - \left(\frac{1}{2}y^2 - 3\right)\right] dy$$

$$= \left(\frac{y^2}{2} + y - \frac{1}{2}\frac{y^3}{3} + 3y\right)\Big|_{-2}^{4}$$

$= $ ______

5. Find the area bounded by $y = 2\cos(3x)$, $y = 2 - 2\cos(3x)$, $x = 0$, $x = \frac{\pi}{3}$.




intersection:

$2\cos(3x) = 2 - 2\cos(3x)$

$4\cos(3x) = 2$

$\cos(3x) = \frac{1}{2}$

$3x = \frac{\pi}{3}$

$\boxed{x = \frac{\pi}{9}}$

$y = 2\cos(3x) <$ $x = \frac{\pi}{3}, y = -2$; $x = 0, y = 2$

$y = 2 - 2\cos(3x) <$ $x = \frac{\pi}{3}, y = 4$; $x = 0, y = 0$

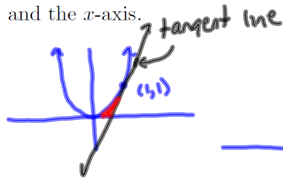
$$A = \int_0^{\frac{\pi}{9}} \left(2\cos(3x) - [2 - 2\cos(3x)]\right) dx + \int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \left[2 - 2\cos(3x) - 2\cos(3x)\right] dx$$

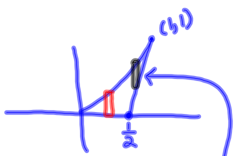
$$A = \int_0^{\frac{\pi}{9}} \left(4\cos(3x) - 2\right) dx + \int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \left[2 - 4\cos(3x)\right] dx$$

$$= \left(4 \cdot \frac{1}{3}\sin(3x) - 2x\right)\Big|_0^{\frac{\pi}{9}} + 2x - 4 \cdot \frac{1}{3}\sin(3x)\Big|_{\frac{\pi}{9}}^{\frac{\pi}{3}}$$

$$= \frac{4}{3}\frac{\sqrt{3}}{2} - \frac{2\pi}{9} - (0) + 2 \cdot \frac{\pi}{3} - \frac{4}{3}(0) - \left(2 \cdot \frac{\pi}{9} - \frac{4}{3}\frac{\sqrt{3}}{2}\right)$$

6. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$ and the $x$-axis.




tangent line: $f(x) = x^2$

$f'(x) = 2x$

$m = f'(1) = 2$

$y - 1 = 2(x - 1) \Rightarrow \boxed{y = 2x - 1}$

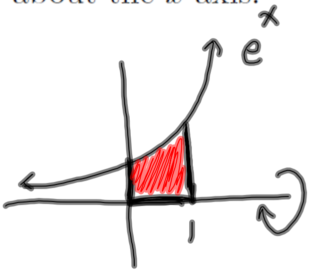
$$A = \int_0^{\frac{1}{2}} x^2\, dx + \int_{\frac{1}{2}}^{1} \left(x^2 - (2x - 1)\right) dx$$

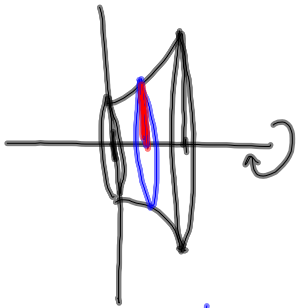
-OR- $A = \int_0^1 \left(\frac{y+1}{2} - \sqrt{y}\right) dy$

$$= \int_0^1 \left(\frac{1}{2}y + \frac{1}{2} - y^{\frac{1}{2}}\right) dy$$

$$= \left(\frac{y^2}{4} + \frac{1}{2}y - \frac{2}{3}y^{\frac{3}{2}}\right)\Big|_0^1$$

7. Find the volume of the solid obtained by revolving the region bounded by $y = e^x$, $y = 0$, $x = 0$, $x = 1$ about the $x$-axis.



cross section is a disk with volume $\pi (r)^2 dx$ $\qquad r = e^x$

$$V = \int_0^1 \pi (e^x)^2 dx = \int_0^1 \pi e^{2x} dx$$

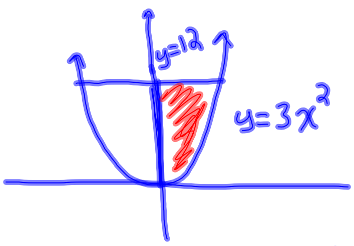
$u = 2x$ ; $x=1, u=2$ ; $x=0, u=0$

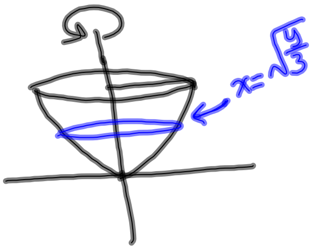
$du = 2dx$

$$= \frac{\pi}{2}\int_0^2 e^u du$$

$$= \frac{\pi}{2} e^u \Big|_0^2 = \frac{\pi}{2}(e^2-1$$

8. Find the volume of the solid obtained by revolving the region bounded by $y = 3x^2$, $y = 12$, $x = 0$ about the $y$-axis.




$$V = \int_0^{12} \pi \left(\sqrt{\frac{y}{3}}\right)^2 dy$$

$$= \pi \int_0^{12} \left(\frac{y}{3}\right) dy$$

$$= \pi \left(\frac{y^2}{6}\right)\Big|_0^{12}$$

9. Find the volume of the solid obtained by revolving the region bounded by $y = x^2$, $y = 4x$, about the $x$-axis, then the $y$ axis.

$x^2 = 4x$

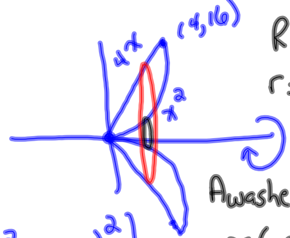
$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0,\ y = 0$

$x = 4,\ y = 16$

① rotate around $x$-axis:




$R = 4x$

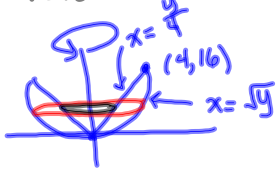
$r = x^2$

$A_{\text{washer}} = \pi(R^2 - r^2)dx$

$$V = \int_0^4 \pi\left[(4x)^2 - (x^2)^2\right]dx$$

$$= \pi\int_0^4 (16x^2 - x^4)\,dx = \underline{\qquad}$$

② rotate around $y$-axis



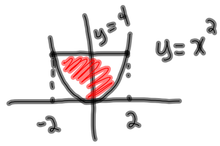

$R = \sqrt{y}$

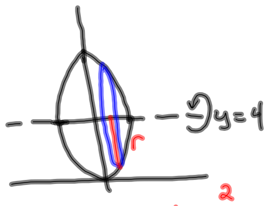
$r = \frac{y}{4}$

$$V = \int_0^{16} \pi\left[(\sqrt{y})^2 - \left(\frac{y}{4}\right)^2\right]dy$$

$$= \pi\int_0^{16}\left(y - \frac{1}{16}y^2\right)dy = \underline{\qquad}$$

#10 R is the region bounded by $y = x^2$, $y = 4$. Revolve around the line $y = 4$.



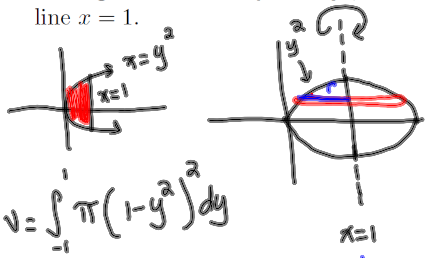

$r = 4 - x^2$

$$V = \int_{-2}^{2} \pi(4 - x^2)^2\,dx$$

OR $\quad 2\int_0^2 \pi(4 - x^2)^2\,dx \quad$ by symmetry

$$2\pi\int_0^2 (16 - 8x^2 + x^4)\,dx = \underline{\qquad}$$

11. Find the volume of the solid obtained by revolving the region bounded by $x = y^2$, $x = 1$, about the line $x = 1$.




$r = 1-y^2$

$$V = \int_{-1}^{1} \pi \left(1-y^2\right)^2 dy$$

or use symmetry $V = 2\int_0^1 \pi (1-y^2)^2 dy$

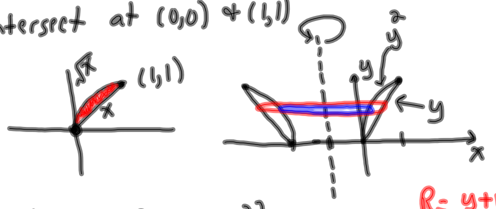
$$= 2\pi \int_0^1 (1-2y^2+y^4) dy$$

$$= \_\_$$

12. Find the volume of the solid obtained by revolving the region bounded by $y = x$, $y = \sqrt{x}$, about the line $x = -1$.

$x = \sqrt{x}$

intersect at $(0,0)$ & $(1,1)$




$R = y+1$

$r = y^2+1$

$$V = \int_0^1 \pi \left[(y+1)^2 - (y^2+1)^2\right] dy$$

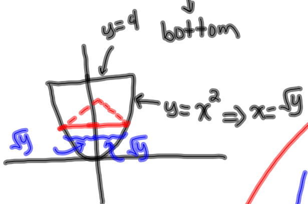
$$= \pi \int_0^1 \left[y^2+2y+1-(y^4+2y^2+1)\right] dy$$

$$= \_\_$$

13. Find the volume of the solid $S$ described here: The base of $S$ is the region bounded by $y = x^2$ and $y = 4$. Cross-sections perpendicular to the $y$ axis are equilateral triangles.

① draw the base of the solid.

bottom




$A_\Delta = \frac{1}{2} bh$


② $V = \int_0^4 (A_\Delta) dy$

$b = 2\sqrt{y}$

$4y = y + h^2$

$3y = h^2$

$h = \sqrt{3y}$

$A_\Delta = \frac{1}{\cancel{2}}(\cancel{2}\sqrt{y})\sqrt{3y}$

$A_\Delta = \sqrt{3}\, y$

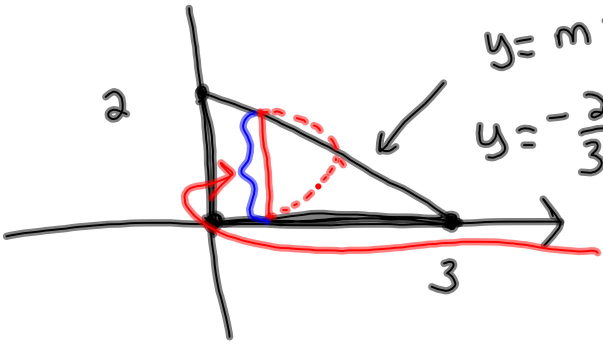
$$= \int_0^4 \sqrt{3}\, y\, dy$$

$$= \frac{\sqrt{3}}{2} y^2 \Big|_0^4$$

$$= \frac{\sqrt{3}}{2}(16) = \boxed{8\sqrt{3}}$$

14. Find the volume of the solid $S$ described here: The base of $S$ is the triangular region with vertices $(0,0)$, $(3,0)$ and $(0,2)$. Cross-sections perpendicular to the ~~semicircles.~~ x-axis are semicircles.

① draw the base




$y = mx + b$ $\quad m = -\frac{2}{3}$

$y = -\frac{2}{3}x + 2$ $\quad b = 2$

diameter of semicircle is $-\frac{2}{3}x + 2$

$A_{\triangle} = \frac{1}{2}\pi r^2$

② $V = \int_0^3 (A_{\triangle})\,dx$

$r = \frac{1}{2}\left(-\frac{2}{3}x + 2\right)$

$r = -\frac{1}{3}x + 1$

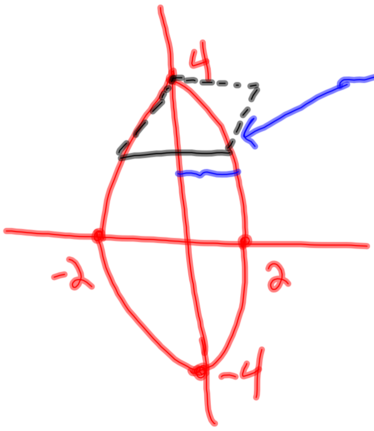
$A_{\triangle} = \frac{1}{2}\pi\left(-\frac{1}{3}x + 1\right)^2$

$$V = \int_0^3 \frac{\pi}{2}\left(-\frac{1}{3}x + 1\right)^2 dx$$

$$= \frac{\pi}{2}\int_0^3 \left(\frac{1}{9}x^2 - \frac{2}{3}x + 1\right)dx = \underline{\qquad}$$

15. Find the volume of the solid $S$ described here: The base of $S$ is the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$. Cross sections perpendicular to the $y$-axis are squares.

① draw base




$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$\frac{x^2}{4} = 1 - \frac{y^2}{16}$$

$$x^2 = 4 - \frac{y^2}{4}$$

$$x = \sqrt{4 - \frac{y^2}{4}}$$

② $V = \int_{-4}^{4} (A_\square)\,dy$ where side $= 2\sqrt{4 - \frac{y^2}{4}}$

$$V = \int_{-4}^{4} \left(2\sqrt{4 - \frac{y^2}{4}}\right)^2 dy$$

$$= \int_{-4}^{4} 4\left(4 - \frac{y^2}{4}\right) dy$$

$$= 4\left(4y - \frac{y^3}{12}\right)\Big|_{-4}^{4} = \_\_\_\_$$