# Fall 2011 Math 151

**Night Before Drill**

*courtesy: Amy Austin*

Review Exercises: Sections 4.3 - 6.1

## Section 4.3

1. Evaluate $\log_3 108 - \log_3 4$
2. Solve for $x$: $\log(x+3) + \log(x) = 1$
3. Solve for $x$: $\ln x - \ln(x+1) = \ln 2 + \ln 3$
4. Find $\lim_{x\to\infty}[\log(2x-1) - \log(3x^2+6)]$
5. What is the domain of $f(x) = \ln(4-x^2)$?

## Section 4.4

6. Find $f'(x)$ for $f(x) = \ln(2x^2-8)$
7. Find the derivative of $f(x) = 2^{\cos x} + \log(3x-1)$
8. Find $y'$ for $y = (\cos x)^{\tan x}$
9. Find $f''(e)$ for $f(x) = \ln(\ln x)$

## Section 4.5

10. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.

    a.) What is the half life of the substance?

    b.) How much of the substance remains after 2.5 years?

11. A bowl of soup at temperature 180° is placed in a 70° room. If the temperature of the soup is 150° after 2 minutes, when will the soup be an eatable 100°?

## Section 4.6

12. Using implicit differentiation, show that $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$.
13. Find the derivative of $y = x^2\cos^{-1}(e^{3x})$
14. Find the equation of the line tangent to $y = \tan^{-1}(2x-1)$ when $x = 1$.
15. Compute the exact value of $\lim_{x\to\infty} \arccos\left(\frac{1+2x}{5-4x}\right)$
16. Compute $\sec(\arctan(-\sqrt{5}))$
17. Compute $\sin^{-1}(\sin\frac{4\pi}{3})$
18. Find the domian of $\arcsin(1-8x^3)$ Where does $\arcsin(1-8x^3)$ cross the $x$ axis.

## Section 4.8

19. Find the limits of each of the following:

    a) $\lim_{x\to 0}\frac{\arcsin(3x)}{2x}$

    b) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x}$

    c.) $\lim_{x\to\infty}\frac{(\ln x)^2}{4x}$

## Section 5.1 - 5.3

20. If $f(x) = \frac{1}{x}$, verify $f(x)$ satisfies the Mean Value Theorem on the interval $[1, 10]$ and find all $c$ that satisfies the conclusion of the Mean Value Theorem.
21. Find the absolute maximum and minimum of the given function on the given interval.

    a) $x^3 - 5x^2 + 3$ on $[-1, 3]$

    b) $x\ln x$ on $[e^{-2}, 1]$

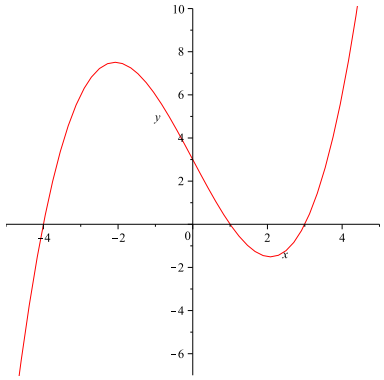
22. Find the intervals where the given function is increasing and decreasing, local extrema, intervals of concavity and inflection points.

    a) $f(x) = x^3 - 2x^2 + x$

    b) $f(x) = x^2e^{2x}$

23. Find the value of $B$ that makes $x = 3$ an inflection point for $y = x^3 + Bx^2 + 4$.

24. In the graph that follows, the graph of $f'$ is given. Using the graph of $f'$, determine all critical values of $f$, where $f$ is increasing and decreasing, local extrema of $f$, where $f$ is concave up and concave down, and the x-coordinates of the inflection points of $f$. Assume $f$ is continuous.




## Section 5.5

25. A cardboard rectangular box holding 32 cubic inches with a square base and open top is to be constructed. If the material for the base costs \$2 per square inch and material for the sides costs \$5 per square inch, find the dimensions of the cheapest such box.

26. Find the shortest distance from the point $(1, 4)$ to the parabola $y^2 = 2x$.

27. The surface area of a closed cylindrical can is 2 square feet. Find the dimensions of the can that maximize the volume of the can.

## Section 5.7

28. Find an antiderivative of $\dfrac{1}{\sqrt{1-x^2}} - \dfrac{1+x}{x}$.

29. Given $f''(x) = 2e^x - 4\sin(x)$, $f(0) = 1$, and $f'(0) = 2$, find $f(x)$.

30. A stone is dropped from a 450 meter tall building.

    a.) Find a formula for the height of the stone at time $t$. Carefully derive the formula you obtain, do not just quote physics formulas. Assume the acceleration due to gravity is $-9.8$ meters per second squared.

    b.) With what velocity does the stone hit the ground?

31. Find the vector functions that describe the velocity and position of a particle that has an acceleration of $\mathbf{a}(t) = \langle 0, 2\rangle$, initial velocity of $\mathbf{v}(0) = \langle 1, -1\rangle$ and an initial position of $\mathbf{r}(0) = \langle 0, 0\rangle$.

## Section 6.1

32. Expand and find the sum: $\sum_{i=2}^{5} i^2$

33. Write $1 + \dfrac{1}{e} + \dfrac{1}{e^2} + \dfrac{1}{e^3} + \dfrac{1}{e^4} + \dfrac{1}{e^5}$ in summation notation.

34. Find the sum: $\sum_{i=2}^{500} (4)$