Fall 2011 Math 151

Night Before Drill
courtesy: Amy Austin

Review Exercises: Sections 4.3 - 6.1

Section 4.3
1. Evaluate \( \log_3 108 - \log_3 4 \)
2. Solve for \( x \): \( \log(x + 3) + \log(x) = 1 \)
3. Solve for \( x \): \( \ln(x - \ln(x + 1)) = \ln 2 + \ln 3 \)
4. Find \( \lim_{x \to \infty} [\log(2x - 1) - \log(3x^2 + 6)] \)
5. What is the domain of \( f(x) = \ln(4 - x^2) \)?

Section 4.4
6. Find \( f'(x) \) for \( f(x) = \ln(2x^2 - 8) \)
7. Find the derivative of \( f(x) = 2^{\cos x} + \log(3x - 1) \)
8. Find \( y' \) for \( y = (\cos x)^\tan x \)
9. Find \( f''(e) \) for \( f(x) = \ln(\ln x) \)

Section 4.5
10. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.
   a.) What is the half life of the substance?
   b.) How much of the substance remains after 2.5 years?
11. A bowl of soup at temperature 180° is placed in a 70° room. If the temperature of the soup is 150° after 2 minutes, when will the soup be an eatable 100°?

Section 4.6
12. Using implicit differentiation, show that \( \frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2} \).
13. Find the derivative of \( y = x^2 \cos^{-1}(e^{3x}) \)
14. Find the equation of the line tangent to \( y = \tan^{-1}(2x - 1) \) when \( x = 1 \).
15. Compute the exact value of \( \lim_{x \to \infty} \arccos \left( \frac{1 + 2x}{5 - 4x} \right) \)
16. Compute \( \sec(\arctan(-\sqrt{3})) \)
17. Compute \( \sin^{-1}(\sin \frac{4\pi}{3}) \)
18. Find the domain of \( \arcsin(1 - 8x^3) \) Where does \( \arcsin(1 - 8x^3) \) cross the \( x \) axis.

Section 4.8
19. Find the limits of each of the following:
   a) \( \lim_{x \to 0} \frac{\arcsin(3x)}{2x} \)
   b) \( \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^{4x} \)
   c) \( \lim_{x \to \infty} \frac{(\ln x)^2}{4x} \)

Section 5.1 - 5.3
20. If \( f(x) = \frac{1}{x} \), verify \( f(x) \) satisfies the Mean Value Theorem on the interval \([1, 10]\) and find all \( c \) that satisfies the conclusion of the Mean Value Theorem.
21. Find the absolute maximum and minimum of the given function on the given interval.
   a) \( x^3 - 5x^2 + 3 \) on \([-1, 3]\)
   b) \( x \ln x \) on \([e^{-2}, 1]\)
22. Find the intervals where the given function is increasing and decreasing, local extrema, intervals of concavity and inflection points.
   a) \( f(x) = x^3 - 2x^2 + x \)
   b) \( f(x) = x^2 e^{2x} \)
23. Find the value of \( B \) that makes \( x = 3 \) an inflection point for \( y = x^3 + Bx^2 + 4 \).
24. In the graph that follows, the graph of \( f' \) is given. Using the graph of \( f' \), determine all critical values of \( f \), where \( f \) is increasing and decreasing, local extrema of \( f \), where \( f \) is concave up and concave down, and the \( x \)-coordinates of the inflection points of \( f \). Assume \( f \) is continuous.

![Graph of f']

Section 5.5

25. A cardboard rectangular box holding 32 cubic inches with a square base and open top is to be constructed. If the material for the base costs $2 per square inch and material for the sides costs $5 per square inch, find the dimensions of the cheapest such box.

26. Find the shortest distance from the point \((1, 4)\) to the parabola \( y^2 = 2x \).

27. The surface area of a closed cylindrical can is 2 square feet. Find the dimensions of the can that maximize the volume of the can.

Section 5.7

28. Find an antiderivative of \( \frac{1}{\sqrt{1-x^2}} - \frac{1+x}{x} \).

29. Given \( f''(x) = 2e^x - 4\sin(x) \), \( f(0) = 1 \), and \( f'(0) = 2 \), find \( f(x) \).

Section 6.1

30. A stone is dropped from a 450 meter tall building.

a.) Find a formula for the height of the stone at time \( t \). Carefully derive the formula you obtain, do not just quote physics formulas. Assume the acceleration due to gravity is \(-9.8 \text{ meters per second squared}\).

b.) With what velocity does the stone hit the ground?

31. Find the vector functions that describe the velocity and position of a particle that has an acceleration of \( \mathbf{a}(t) = (0, 2) \), initial velocity of \( \mathbf{v}(0) = (1, -1) \) and an initial position of \( \mathbf{r}(0) = (0, 0) \).

32. Expand and find the sum: \( \sum_{i=2}^{5} i^2 \)

33. Write \( 1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5} \) in summation notation.

34. Find the sum: \( \sum_{i=2}^{500} (4) \)