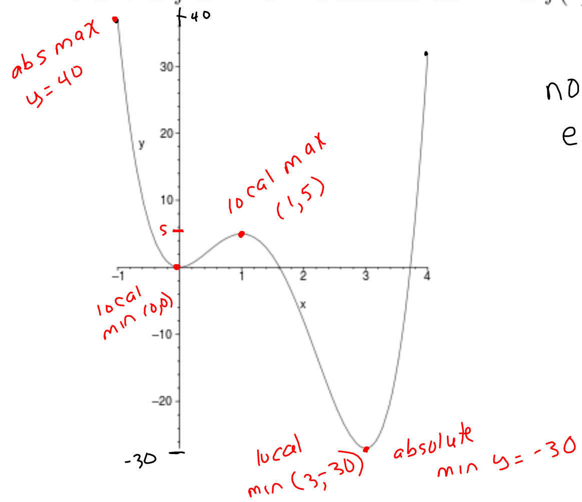
Definition

(1) A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. A function f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. In this case, we call $f(c)$ the **maximum value** or **minimum value**, respectively.

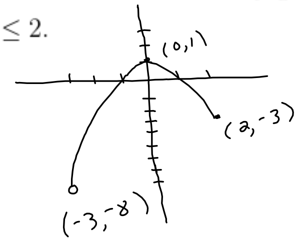
(2) A function f has a **local maximum** at c if $f(c) \geq f(x)$ when x is near c. A function f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.



no local extrema at endpoints.

EXAMPLE 1: Find all absolute and local extrema by graphing the function:

(a) $f(x) = 1 - x^2$, $-3 < x \leq 2$.



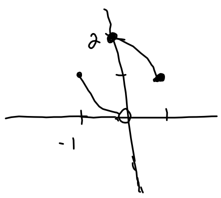
abs max: y=1

abs min: none

local max: (0,1)

local min: none

(b) $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ 2 - x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$



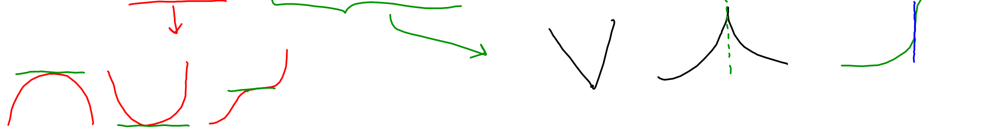
abs max y=2

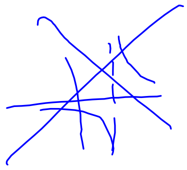
abs min none

local max (0,2)

local min none

Definition We call $x = c$ a **critical number** of $f(x)$ if $x = c$ is in the domain of f and either $f'(c) = 0$ or $f'(c)$ does not exist.





EXAMPLE 2: Find the domain and critical values for the following functions:

(a) $f(x) = 4x^3 - 9x^2 - 12x + 3$ doman $\mathbb{R}$

$f'(x) = 12x^2 - 18x - 12$

$= 6(2x^2 - 3x - 2) \to 6(2x+1)(x-2) = 0$

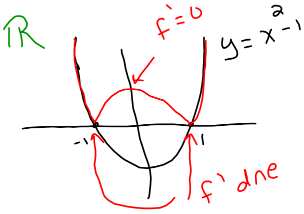
$2x + 1 = 0$ $\boxed{x = -\frac{1}{2}}$

$x - 2 = 0$ $\boxed{x = 2}$

(b) $f(x) = |x^2 - 1|$ domain: $\mathbb{R}$

~~$f'(x) = |2x|$~~

~~cn: $x = 0$~~



cn: $x = 0$ because $f'(0) = 0$

cn: $x = \pm 1$ because $f'(\pm 1)$ dne

(c) $f(x) = \sqrt[3]{x^2 - 3x}$ domain: $\mathbb{R}$

$f(x) = (x^2 - 3x)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(x^2 - 3x)^{-\frac{2}{3}}(2x - 3)$

$f'(x) = \dfrac{2x - 3}{3(\underbrace{x^2 - 3x}_{x(x-3)})^{2/3}}$

cn: $f'(x) = 0 \to 2x - 3 = 0$ $\boxed{x = \frac{3}{2}}$

cn: $f'(x)$ dne $x = 0$ $x = 3$

(d) $f(x) = xe^{2x}$ domain $\mathbb{R}$

$f'(x) = e^{2x} + 2xe^{2x}$

$= e^{2x}(1 + 2x)$

never zero

equals zero when $\boxed{x = -\frac{1}{2}}$

(e) $f(x) = x \ln x$ domain: $(0, \infty)$

$f'(x) = \ln x + x\left(\frac{1}{x}\right)$

$f'(x) = \ln x + 1$

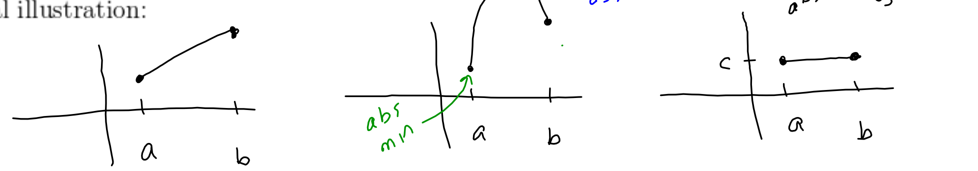
$\ln x + 1 = 0 \to \ln x = -1$

$\boxed{x = e^{-1}}$

Extreme Value Theorem If $f(x)$ is a continuous function on a closed interval $[a, b]$, then f will attain both an absolute maximum and an absolute minimum.

endpoints included.

Graphical illustration:



EXAMPLE 3: Use the extreme value theorom to find the absolute extrema for $f(x) = 1 + 27x - x^3$ on the interval $[0, 4]$.

step 1: Find cn that lie in $\boxed{[0,4]}$

$f'(x) = 27 - 3x^2$

$= 3(9 - x^2)$

cn : $\boxed{x = 3}$ only

since $x = -3$ not in interval.

$27 \cdot 3 = 81$

$f(3) = 1 + 81 - 27$

$= 82 - 27$

$f(3) = \boxed{55}$

$f(0) = \boxed{1}$

$f(4) = 1 + 108 - 64$

$= 109 - 64$

$= \boxed{45}$

$\boxed{\begin{array}{l} \text{abs max} = 55 \\ \text{abs min} = 1 \end{array}}$

EXAMPLE 4: Use the extreme value theorom to find the absolute extrema for $f(x) = x - 2\cos x$ on the interval $[0, \pi]$.

$f'(x) = 1 + 2\sin x$

$f'(x) = 0 \quad 1 + 2\sin x = 0$

$\sin x = -\frac{1}{2}$ never happens in $[0, \pi]$

no cn

$f(0) = -2 \leftarrow$ abs min

$f(\pi) = \pi + 2 \leftarrow$ abs max