

First, Go back to section 11.2

9. Consider $\sum_{n=1}^{\infty} (x-5)^n$. Find the value(s) of x for which the series converges. Find the sum of the series for those values of x .

Recall
$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad \text{if } |r| < 1$$

$= \frac{\text{"First term"}}{1-r}$

$r = x-5 \rightarrow |x-5| < 1$

$-1 < x-5 < 1$

$4 < x < 6$

Interval notation $(4, 6)$

Sum of $\sum_{n=1}^{\infty} (x-5)^n = \sum_{n=1}^{\infty} \underbrace{(x-5)}_a \underbrace{(x-5)}_r = \frac{a}{1-r}$

or
$$\frac{\text{"First term"}}{1-r} = \frac{x-5}{1-(x-5)} = \frac{x-5}{1-(x-5)}$$

$= \frac{x-5}{6-x}$ Sum!

$a^n b^n = (ab)^n$

$$\sum_{n=2}^{\infty} \underbrace{(-3)}_a \left(\frac{2}{7}\right)^{n-1} = \sum_{n=2}^{\infty} (-3)(-3)^{n-1} \left(\frac{2}{7}\right)^{n-1}$$

$$\frac{\text{First term}}{1-r} = \frac{9\left(\frac{2}{7}\right)}{1 + \frac{6}{7}} = \sum_{n=2}^{\infty} (-3) \left(\frac{-6}{7}\right)^{n-1}$$

$= \sum_{n=2}^{\infty} \underbrace{(-3)}_a \underbrace{\left(\frac{-6}{7}\right)}_r^{n-1}$

$= \frac{\frac{18}{7}}{1 + \frac{6}{7}}$

Remainder estimate for integral test

consider $\sum_{n=1}^{\infty} a_n = \underbrace{a_1 + a_2 + \dots + a_n}_{S_n} + \underbrace{a_{n+1} + a_{n+2} + \dots}_{\text{remainder, } R_n}$

we can use $S_n = a_1 + a_2 + \dots + a_n$ to estimate the sum of the series.

$R_n < \int_n^{\infty} f(x) dx$, where $f(n) = a_n$

consider $\sum_{n=1}^{\infty} \frac{8}{n^4}$. use S_9 to approximate the sum and use the remainder estimate for the integral test to estimate the remainder (error).

$S_9 = a_1 + a_2 + \dots + a_9$

$S_9 = 8 + \frac{8}{2^4} + \frac{8}{3^4} + \dots + \frac{8}{9^4}$

$R_9 < \frac{8}{3(9^3)}$

$R_9 < \int_9^{\infty} \frac{8}{x^4} dx$

$\lim_{t \rightarrow \infty} \int_9^t \frac{8}{x^4} dx$

$\lim_{t \rightarrow \infty} \left. -\frac{8}{3x^3} \right|_9^t$

$\lim_{t \rightarrow \infty} \left[\cancel{-\frac{8}{3t^3}} + \frac{8}{3(9^3)} \right] = \frac{8}{3(9^3)}$

use the remainder estimate for the integral test to find the value of n that will ensure S_n approximates

$\sum_{n=1}^{\infty} \frac{3}{n^3}$ to within $\frac{1}{100}$. $\rightarrow R_n < \frac{1}{100}$

$R_n < \int_n^{\infty} \frac{3}{x^3} dx < \frac{1}{100}$

$\lim_{t \rightarrow \infty} \int_n^t \frac{3}{x^3} dx = \lim_{t \rightarrow \infty} \left. -\frac{3}{2x^2} \right|_n^t$

$= \lim_{t \rightarrow \infty} \left(\cancel{-\frac{3}{2t^2}} + \frac{3}{2n^2} \right)$

$= \frac{3}{2n^2} < \frac{1}{100}$

$\frac{300}{2} < n^2$

$150 < n^2$ $n=13$

Given a series $\sum a_n$

Test For divergence

$\lim_{n \rightarrow \infty} a_n \neq 0$

→ diverges

If $\lim_{n \rightarrow \infty} a_n = 0$

Sum of series

Just asked if

$\sum_{n=1}^{\infty} a_n$ converges

$a_n > 0$ & decreasing
use integral test

geometric

$\sum ar^n$

telescoping

given

a formula

for $S_n \rightarrow \sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} S_n$

Does

$\sum_{n=1}^{\infty} \frac{e^n}{\sqrt{n}}$

converge or diverge?

T.D

$\lim_{n \rightarrow \infty} \frac{e^{2n}}{\sqrt{n}} = \frac{\infty}{\infty} \stackrel{L}{=} \lim_{n \rightarrow \infty} \frac{e^{\sqrt{n}} \cdot \frac{1}{2\sqrt{n}}}{\frac{1}{2\sqrt{n}}} = \infty \neq 0$

diverges by T.D

② $\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+2}} \right)$

$n + n+2$

are not consecutive → include "a_{n-1}" in S_n

Telescoping series!

~~$S_n = a_1, a_2, a_3, \dots, a_{n-1}, a_n$~~

$S_n = a_1 + a_2 + \dots + a_{n-1} + a_n$

$S_n = e - e^{\frac{1}{3}} + e^{\frac{1}{2}} - e^{\frac{1}{4}} + e^{\frac{1}{3}} - e^{\frac{1}{5}} + e^{\frac{1}{4}} - e^{\frac{1}{6}} + \dots + e^{\frac{1}{n}} - e^{\frac{1}{n+1}} + e^{\frac{1}{n+1}} - e^{\frac{1}{n+2}}$

$S_n = e + e^{\frac{1}{2}} - e^{\frac{1}{n+1}} - e^{\frac{1}{n+2}}$

$\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+2}} \right) = \lim_{n \rightarrow \infty} \left(e + e^{\frac{1}{2}} - e^{\frac{1}{n+1}} - e^{\frac{1}{n+2}} \right) = e + e^{\frac{1}{2}} - 2$

$\sum_{n=8}^{\infty} \frac{4}{n(n+1)} =$ PF $\frac{4}{n(n+1)}$

$\Rightarrow \sum_{n=8}^{\infty} \left(\frac{4}{n} - \frac{4}{n+1} \right)$
Telescope!

$\frac{4}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$4 = A(n+1) + Bn$

$n=0 \rightarrow A=4$

$n=-1 \rightarrow B=-4$