Section 11.11 Taylor Polynomials

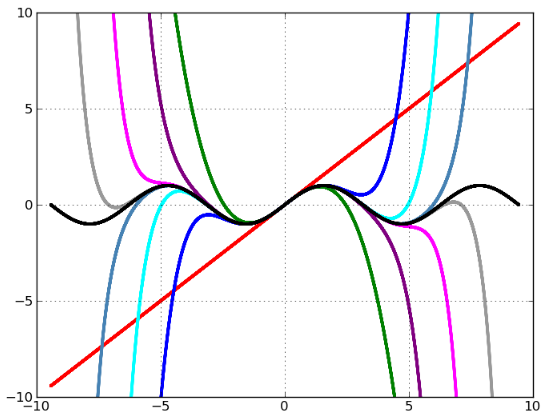
Definition: Let $f(x)$ be a function. Recall the Taylor Series for $f(x)$ at $x = a$ is

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. A partial sum of a Taylor Series is called a Taylor Polynomial. More specifically, the n^{th} degree **Taylor Polynomial** for $f(x)$ at $x = a$ is

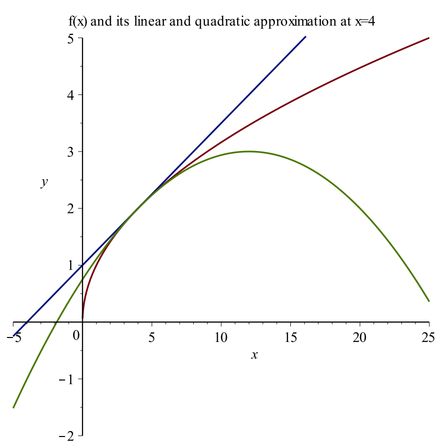
$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

where $f^{(i)}(a)$ is the ith derivative of $f(x)$ at $x = a$.

The Taylor polynomial at $x = a$ is useful in approximating a function near $x = a$. The first degree Taylor polynomial at $x = a$ is the same as the tangent line to $f(x)$ at $x = a$, and the second degree Taylor polynomial at $x = a$ is the same as the quadratic approximation to $f(x)$ at $x = a$. The higher the degree, the better approximation.



1. Find both the first and second degree Taylor polynomial for $f(x) = \sqrt{x}$ at $x = 4$.



2. Find the third degree Taylor Polynomial for $f(x) = xe^x$ at $x = 2$.

3. Find $T_8(x)$ for $f(x) = x^2 e^{-2x^3}$ at $x = 0$.