MATH 152
SPRING 2019

Sample Exam (covering sections 5.5-7.2)

1. Find the area of the region bounded by $y = x^3$, $y = x$ from $x = 0$ to $x = 2$.

a) $\frac{3}{2}$

b) 2

c) $\frac{1}{2}$

d) $\frac{5}{2}$

e) 3

2. If we revolve the region bounded by $x = 2y^2$ and $x = 2$ about the line $x = 2$, which of the following integrals gives the resulting volume?

a) $\int_{-1}^{1} \pi(4 - 4y^4)\, dy$

b) $\int_{-1}^{1} \pi(4 - (2 - 2y^2)^2)\, dy$

c) $\int_{-1}^{1} 4\pi y^4\, dy$

d) $\int_{-1}^{1} \pi(2 - 2y^2)^2\, dy$

e) $\int_{-1}^{1} \pi(4y^4 - 4)\, dy$

3. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

a) $\frac{75}{4}$ J

b) 45 J

c) $\frac{75}{2}$ J

d) 30 J

e) 40 J

4. Evaluate $\int_0^{\sqrt[3]{\pi/2}} x^5 \cos(x^3)\, dx$

a) $\frac{\pi}{6} - \frac{1}{3}$

b) $\frac{\pi}{3} - \frac{1}{6}$

c) $\frac{\pi}{2} - \frac{1}{3}$

d) $\frac{\pi}{3} - \frac{1}{2}$

e) $\frac{\pi}{6} - \frac{1}{2}$

5. $\int_1^{e^4} x \ln x\, dx =$

a) $\frac{7e^8 + 1}{4}$

b) $\frac{9e^8 + 1}{4}$

c) $\frac{8e^8 + 1}{4}$

d) $\frac{7e^8 - 1}{4}$

e) $\frac{8e^8 - 1}{4}$

6. $\int \sin^2(x)\, dx =$

a) $\frac{x}{2} + \frac{1}{4}\sin(2x) + C$

b) $\frac{x}{2} - \frac{1}{4}\sin(2x) + C$

c) $\frac{4}{3}\sin^3(x) + C$

d) $\frac{x}{2} + 2\sin(2x) + C$

e) $\frac{1}{3}\sin^3(x) + C$

7. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling $\frac{1}{3}$ of this rope to the top of the cliff?

a) 125 foot-pounds

b) 25 foot-pounds

c) 35 foot-pounds

d) 2255 foot-pounds

e) 75 foot-pounds

8. $\int_0^{\pi/4} \sec^4 x \tan^2 \, dx$

a) $\frac{16}{3}$

b) $\frac{4}{3}$

c) $\frac{8}{3}$

d) $\frac{1}{6}$

e) $\frac{8}{15}$e

9. $\int \frac{x}{(x-1)^2} \, dx$

a) $\ln|x-1| + \frac{1}{x-1} + C$

b) $\ln|x-1| - \frac{1}{x-1} + C$

c) $\ln|x-1| + \frac{1}{3(x-1)^2} + C$

d) $\ln|x-1| - \frac{1}{3(x-1)^2} + C$

e) $\ln|x-1| + \frac{3}{(x-1)^2} + C$

Part II - Work Out Problems

10. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the x-axis.

11. The base of a solid is the region bounded by $y = x^2$ and $y = 1$. Cross-sections perpendicular the the y-axis are semi-circles. Set up but do not evaluate an integral that gives the volume of the solid.

12. A 15 m long trough with semicircular ends of radius 2 m is full of water. Set up but **do not evaluate** an integral that will compute the work required to pump all of the water out of a 1 m high spout. **Indicate on the picture where you are placing the axis and which direction is positive.** Note: The density of water is $\rho = 1000\ kg/m^3$ and the acceleration due to gravity is $9.8\ m/s^2$.

13. Using cylindrical shells, set up but do not evaluate an integral that gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.

14. Consider the region R bounded by $y = \sqrt{x} + 3$, $y = 3$, $x = 16$

a.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the x-axis

b.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the y-axis

c.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $x = -1$

d.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $y = 10$.

15. Find $\int \sec^5 x \tan^3 x \, dx$.

16. Find $\int \sin^5(3x) \cos^2(3x) \, dx$.

17. Evaluate $\int \arccos x \, dx$.

17. Evaluate $\int e^x \cos(2x)\, dx$.