

MATH 152
SPRING 2019

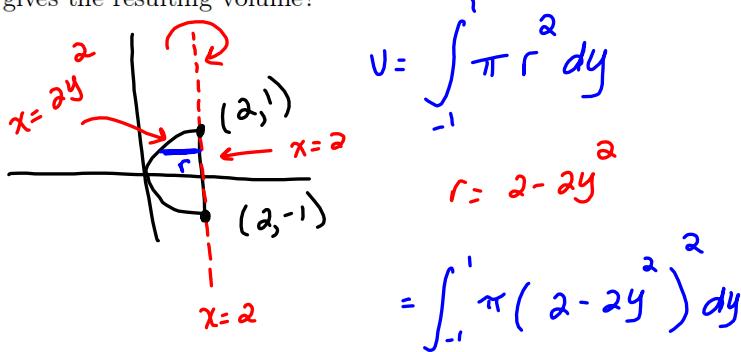
Sample Exam (covering sections 5.5-7.2)

1. Find the area of the region bounded by $y = x^3$, $y = x$ from $x = 0$ to $x = 2$.

- a) $\frac{3}{2}$
- b) 2
- c) $\frac{1}{2}$
- d) $\frac{5}{2}$
- e) 3

2. If we revolve the region bounded by $x = 2y^2$ and $x = 2$ about the line $x = 2$, which of the following integrals gives the resulting volume?

- a) $\int_{-1}^1 \pi(4 - 4y^4) dy$
- b) $\int_{-1}^1 \pi(4 - (2 - 2y^2)^2) dy$
- c) $\int_{-1}^1 4\pi y^4 dy$
- d) $\int_{-1}^1 \pi(2 - 2y^2)^2 dy$
- e) $\int_{-1}^1 \pi(4y^4 - 4) dy$



3. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

a) $\frac{75}{4}$ J

b) 45 J

c) $\frac{75}{2}$ J

d) 30 J

e) 40 J

$$f(x) = kx$$

Given $f(1) = 10$

$$k(1) = 10$$

$$f(x) = 10x$$

$$\begin{aligned} W &= \int_1^3 10x \, dx \\ &= 5x^2 \Big|_1^3 \end{aligned}$$

$$k = 10$$

4. Evaluate $\int_0^{\sqrt[3]{\pi/2}} x^5 \cos(x^3) \, dx$

a) $\frac{\pi}{6} - \frac{1}{3}$

b) $\frac{\pi}{3} - \frac{1}{6}$

c) $\frac{\pi}{2} - \frac{1}{3}$

d) $\frac{\pi}{3} - \frac{1}{2}$

e) $\frac{\pi}{6} - \frac{1}{2}$

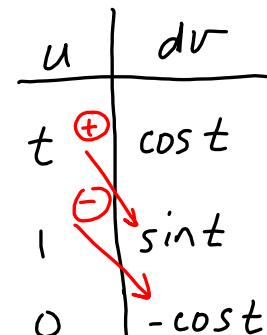
$$\downarrow x^2 x^3$$

$$\frac{1}{3} \int_0^{\frac{\pi}{2}} t \cos t \, dt$$

$$\frac{1}{3} \left(t \sin t + \cos t \right) \Big|_0^{\frac{\pi}{2}}$$

$$t = x^3 \quad \begin{cases} x = \sqrt[3]{\frac{\pi}{2}}, & t = \frac{\pi}{2} \\ x = 0, & t = 0 \end{cases}$$

$$dt = 3x^2 \, dx$$



$$\frac{1}{3} \left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} - (0 + \cos 0) \right)$$

$$\frac{1}{3} \left(\frac{\pi}{2} - 1 \right)$$

5. $\int_1^{e^4} x \ln x \, dx =$ parts $u = \ln x$ $dv = x \, dx$

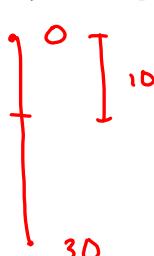
a) $\frac{7e^8 + 1}{4}$
 b) $\frac{9e^8 + 1}{4}$
 c) $\frac{8e^8 + 1}{4}$
 d) $\frac{7e^8 - 1}{4}$
 e) $\frac{8e^8 - 1}{4}$

6. $\int \sin^2(x) \, dx =$ $\int \frac{1}{2} (1 - \cos 2x) \, dx$

a) $\frac{x}{2} + \frac{1}{4} \sin(2x) + C$
 b) $\frac{x}{2} - \frac{1}{4} \sin(2x) + C$
 c) $\frac{4}{3} \sin^3(x) + C$
 d) $\frac{x}{2} + 2 \sin(2x) + C$
 e) $\frac{1}{3} \sin^3(x) + C$

7. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling $\frac{1}{3}$ of this rope to the top of the cliff?

- a) 125 foot-pounds
- b) 25 foot-pounds
- c) 35 foot-pounds
- d) 2255 foot-pounds
- e) 75 foot-pounds



$$W = \int \left(\text{Total weight} - \frac{\text{weight of rope}}{\text{per unit}} x \right) dx$$

$$= \int_0^{10} \left(15 - \frac{1}{2} x \right) dx$$

$$8. \int_0^{\pi/4} \sec^4 x \tan^2 x \, dx$$

- a) $\frac{16}{3}$
- b) $\frac{4}{3}$
- c) $\frac{8}{3}$
- d) $\frac{1}{6}$
- e) $\frac{8}{15}$

$$\int_0^{\frac{\pi}{4}} \sec^2 x \tan^2 x \sec^2 x \, dx$$

$u = \tan x \quad x = 0, u = 0$

$x = \frac{\pi}{4}, u = 1$

$$du = \sec^2 x \, dx$$

$$\int_0^1 (u^2 + 1) u^2 \, du$$

9. $\int \frac{x}{(x-1)^2} dx$
- a) $\ln|x-1| + \frac{1}{x-1} + C$
 b) $\ln|x-1| - \frac{1}{x-1} + C$
 c) $\ln|x-1| + \frac{1}{3(x-1)^2} + C$
 d) $\ln|x-1| - \frac{1}{3(x-1)^2} + C$
 e) $\ln|x-1| + \frac{3}{(x-1)^2} + C$

$$u = x-1 \rightarrow x = u+1$$

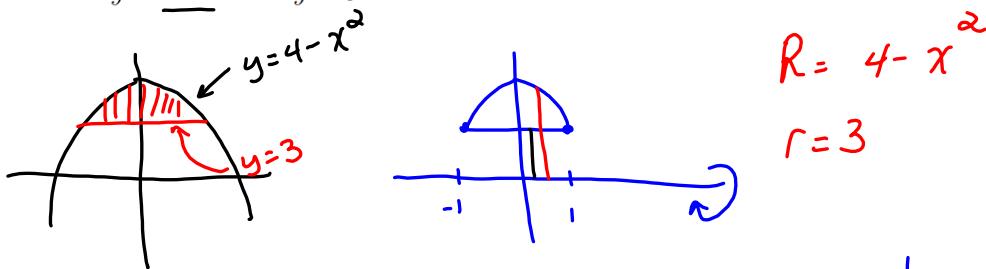
$$du = dx$$

$$\int \frac{u+1}{u^2} du = \int \frac{1}{u} + \frac{1}{u^2} du$$

$$= \ln|u| - \frac{1}{u} + C$$

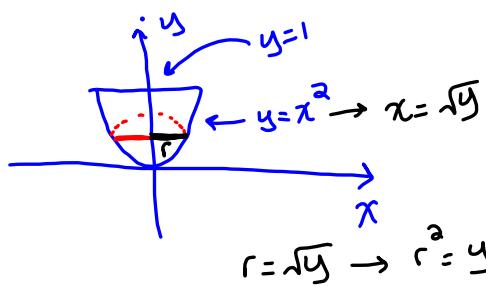
Part II - Work Out Problems

10. Find the volume of the solid obtained by revolving the region bounded by $y = \underline{4 - x^2}$ and $y = 3$ about the x -axis.



$$V = \int_{-1}^1 \pi (R^2 - r^2) dx = 2 \int_0^1 \pi ((4-x^2)^2 - 9) dx$$

11. The base of a solid is the region bounded by $y = x^2$ and $y = 1$. Cross-sections perpendicular to the y -axis are semi-circles. Set up but do not evaluate an integral that gives the volume of the solid.



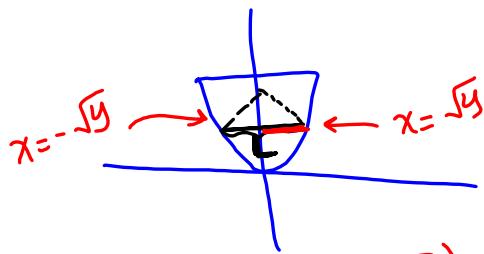
$$V = \int_0^1 \text{Cross-section} dy$$

\downarrow

$$\frac{1}{2}\pi r^2$$

Get r in terms of y

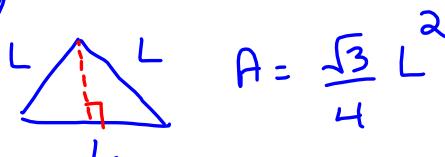
$$V = \int_0^1 \frac{1}{2}\pi(r(y))^2 dy$$



$$L = R - L = \sqrt{y} - (-\sqrt{y})$$

$$\text{or } L = 2\sqrt{y}$$

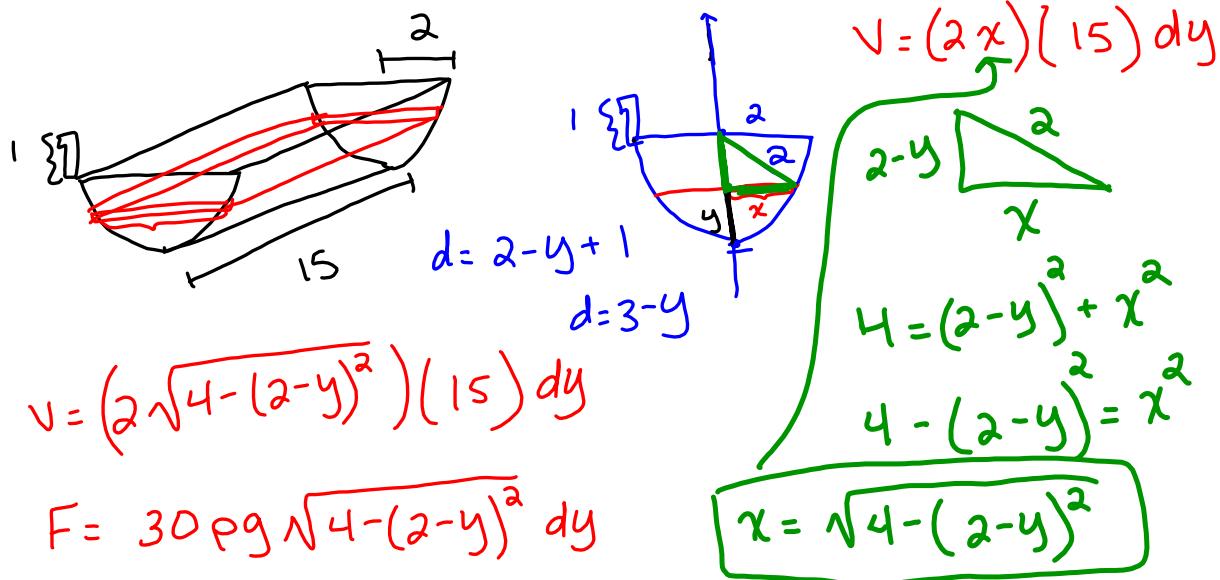
cross-sections are equilateral triangles



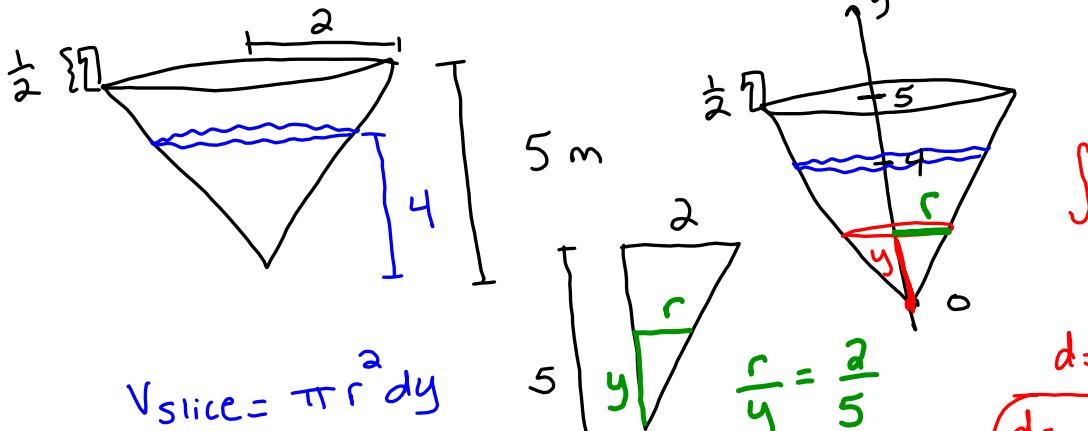
$$A = \frac{\sqrt{3}}{4} L^2$$

$$V = \int_0^1 \frac{\sqrt{3}}{4} L^2 dy = \int_0^1 \frac{\sqrt{3}}{4} (2\sqrt{y})^2 dy$$

12. A 15 m long trough with semicircular ends of radius 2 m is full of water. Set up but do not evaluate an integral that will compute the work required to pump all of the water out of a 1 m high spout. [Indicate on the picture where you are placing the axis and which direction is positive] Note: The density of water is $\rho = 1000 \text{ kg/m}^3$ and the acceleration due to gravity is 9.8 m/s^2 .



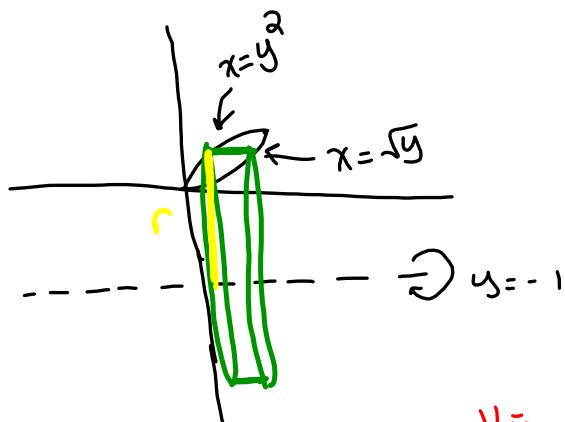
$$W = \int_0^2 30\rho g (3-y) \sqrt{4-(2-y)^2} dy$$



$$V_{slice} = \pi \left(\frac{2}{5}y\right)^2 dy$$

$$W = \int_0^4 \pi \frac{4}{25}y^2 \rho g \left(\frac{11}{2}-y\right) dy$$

13. Using cylindrical shells, set up but do not evaluate an integral that gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.



$$V = \int 2\pi r h dy$$

$$r = y - (-1) = y + 1$$

$$h = \sqrt{y} - y^2$$

$$V = \int_0^1 2\pi (y+1)(\sqrt{y} - y^2) dy$$

14. Consider the region R bounded by $y = \sqrt{x} + 3$, $y = 3$, $x = 16$
- Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the x -axis
 - Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the y -axis
 - Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $x = -1$
 - Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $y = 10$.

15. Find $\int \sec^5 x \tan^3 x dx$.

$$\begin{aligned} &= \int \underbrace{\sec^4 x}_{u^4} \underbrace{\tan^3 x}_{\sec^2 x - 1} \underbrace{\sec x \tan x dx}_{du} \\ u &= \sec x \\ du &= \sec x \tan x dx \\ &\int u^4 (u^2 - 1) du = \int (u^6 - u^4) du \\ &= \frac{\sec x}{7} - \frac{\sec^5 x}{5} + C \end{aligned}$$

16. Find $\int \sin^5(3x) \cos^2(3x) dx$.

$$\begin{aligned} &= \int \underbrace{\sin^4(3x)}_{(\sin^2(3x))^2} \underbrace{\cos^2(3x)}_{u^2} \underbrace{\sin(3x) dx}_{-\frac{du}{3}} \\ u &= \cos(3x) \\ du &= -3 \sin(3x) dx \\ &\quad (\sin^2(3x)) \\ &\quad \downarrow \\ &\quad (1 - \cos^2(3x))^2 \\ &\quad \downarrow \\ &\quad (1 - u^2)^2 \end{aligned}$$

$$\begin{aligned} &\quad \frac{-1}{3} \int (1 - u^2)^2 u^2 du \\ &\quad \text{Foil} \end{aligned}$$

17. Evaluate $\int \arccos x dx$.

Parts!

$$\begin{aligned} u &= \arccos x & dv &= dx \\ du &= -\frac{1}{\sqrt{1-x^2}} & v &= x \end{aligned}$$

$$\int \arccos x dx = uv - \int v du$$

$$= x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$x \arccos x - \sqrt{1-x^2} + C$$

$$\begin{aligned} &\text{u-sub!} \\ &u = 1 - x^2 \\ &du = -2x dx \\ &-\frac{1}{2} \int u^{-\frac{1}{2}} du = -\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \\ &= -\sqrt{u} \end{aligned}$$

18

Evaluate $\int e^x \cos(2x) dx$.

$$u = \cos(2x)$$

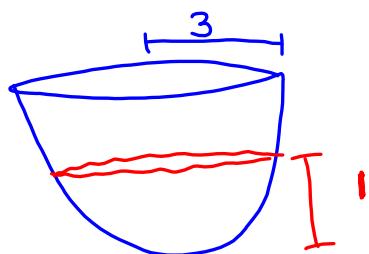
$$dv = e^x dx$$

$$du = -2\sin(2x)dx$$

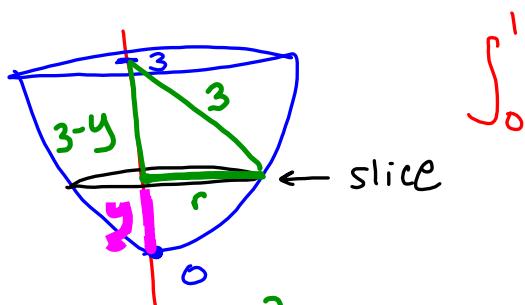
$$v = e^x$$

$$\begin{aligned} \int e^x \cos(2x) dx &= e^x \cos(2x) + \boxed{\int 2e^x \sin(2x) dx} \\ &\quad u = 2\sin(2x) \quad dv = e^x dx \\ &\quad du = 4\cos(2x)dx \quad \cancel{-} \quad v = e^x \\ &= e^x \cos(2x) + 2e^x \sin(2x) - \int 4e^x \cos(2x) dx \end{aligned}$$

$$\cancel{\int} \int e^x \cos(2x) dx = \boxed{\frac{e^x \cos(2x) + 2e^x \sin(2x)}{5} + C}$$



Partially full hemisphere!



$$V = \pi r^2 dy$$

$$9 = (3-y)^2 + r^2 \rightarrow r^2 = 9 - (3-y)^2 \quad d = 3-y$$

$$V = \pi (9 - (3-y)^2) dy$$

$$W = \int_0^1 \pi (9 - (3-y)^2) \rho g (3-y) dy$$

