

MATH 152
SPRING 2019

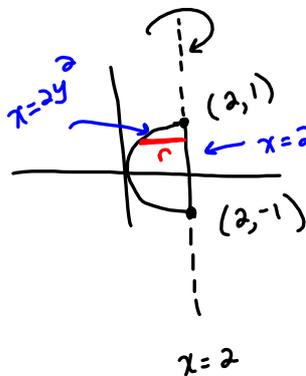
Sample Exam (covering sections 5.5-7.2)

1. Find the area of the region bounded by $y = x^3$, $y = x$ from $x = 0$ to $x = 2$.

- a) $\frac{3}{2}$
- b) 2
- c) $\frac{1}{2}$
- d) $\frac{5}{2}$
- e) 3

2. If we revolve the region bounded by $x = 2y^2$ and $x = 2$ about the line $x = 2$, which of the following integrals gives the resulting volume?

- a) $\int_{-1}^1 \pi(4 - 4y^4) dy$
- b) $\int_{-1}^1 \pi(4 - (2 - 2y^2)^2) dy$
- c) $\int_{-1}^1 4\pi y^4 dy$
- d) $\int_{-1}^1 \pi(2 - 2y^2)^2 dy$
- e) $\int_{-1}^1 \pi(4y^4 - 4) dy$



$$V = \int_{-1}^1 \pi r^2 dy$$

$$x = 2 \rightarrow 2 = 2y^2$$

$$y^2 = 1$$

$$y = \pm 1$$

$$r = R - L \\ = 2 - 2y^2$$

$$V = \int_{-1}^1 \pi (2 - 2y^2)^2 dy$$

3. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

- a) $\frac{75}{4}$ J
- b) 45 J
- c) $\frac{75}{2}$ J
- d) 30 J
- e) 40 J

$f(x) = kx$ $f = \text{force} = 10$
 $10 = k(1)$ $x = \text{units beyond natural length}$
 $k = 10$ $f(1) = 10$
 $f(x) = 10x$
 $W = \int_1^3 10x dx = 40 \text{ J}$

4. Evaluate $\int_0^{\sqrt[3]{\pi/2}} x^5 \cos(x^3) dx$

- a) $\frac{\pi}{6} - \frac{1}{3}$
- b) $\frac{\pi}{3} - \frac{1}{6}$
- c) $\frac{\pi}{2} - \frac{1}{3}$
- d) $\frac{\pi}{3} - \frac{1}{2}$
- e) $\frac{\pi}{6} - \frac{1}{2}$

$x^2 \cdot x^3$

$t = x^3 \begin{cases} x = \sqrt[3]{\frac{\pi}{2}}, t = \frac{\pi}{2} \\ x = 0, t = 0 \end{cases}$
 $dt = 3x^2 dx$
 $x^2 dx = \frac{dt}{3}$

$\frac{1}{3} \int_0^{\frac{\pi}{2}} t \cos t dt$

u	dv
t ⊕	cost
1 ⊖	sint
0	-cost

$\frac{1}{3} (t \sin t + \cos t) \Big|_0^{\frac{\pi}{2}}$

$\frac{1}{3} (\frac{\pi}{2} - 1)$

5. $\int_1^{e^4} x \ln x \, dx =$

a) $\frac{7e^8 + 1}{4}$

b) $\frac{9e^8 + 1}{4}$

c) $\frac{8e^8 + 1}{4}$

d) $\frac{7e^8 - 1}{4}$

e) $\frac{8e^8 - 1}{4}$

u	dv
$\ln x$ \oplus	x

$\frac{1}{x} \xrightarrow{-\int} \frac{x^2}{2}$

$\frac{x^2}{2} \ln x \Big|_1^{e^4} - \int_1^{e^4} \frac{x^2}{2} \cdot \frac{1}{x} \, dx$

$\left(\frac{x^2}{2} \ln x - \frac{1}{4} x^2 \right) \Big|_1^{e^4}$

$$\frac{e^8}{2} \ln e^4 - \frac{e^8}{4} - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right)$$

$$\frac{e^8}{2} (4) - \frac{e^8}{4} + \frac{1}{4}$$

6. $\int \sin^2(x) \, dx = \int \frac{1}{2} (1 - \cos 2x) \, dx$

a) $\frac{x}{2} + \frac{1}{4} \sin(2x) + C$

b) $\frac{x}{2} - \frac{1}{4} \sin(2x) + C$

c) $\frac{4}{3} \sin^3(x) + C$

d) $\frac{x}{2} + 2 \sin(2x) + C$

e) $\frac{1}{3} \sin^3(x) + C$

$$\frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + C$$

7. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling $\frac{1}{3}$ of this rope to the top of the cliff?

- a) 125 foot-pounds
- b) 25 foot-pounds
- c) 35 foot-pounds
- d) 2255 foot-pounds
- e) 75 foot-pounds



$$\int \left[\text{Total weight} - \left(\frac{\text{weight of rope}}{\text{per unit}} \right) x \right] dx$$

$$W = \int_0^{10} \left(15 - \frac{1}{2} x \right) dx$$

$$= \left(15x - \frac{x^2}{4} \right) \Big|_0^{10}$$

or

$$\int_0^{10} \frac{1}{2} x dx + (20) \left(\frac{1}{2} \right) (10)$$

8. $\int_0^{\pi/4} \sec^4 x \tan^2 x dx =$

- a) $\frac{16}{3}$
- b) $\frac{4}{3}$
- c) $\frac{8}{3}$
- d) $\frac{1}{6}$
- e) $\frac{8}{15}$

$$\int_0^{\pi/4} \sec^2 x \tan^2 x \sec^2 x dx$$

$u = \tan x$ $\left\{ \begin{array}{l} x = \frac{\pi}{4}, u = 1 \\ x = 0, u = 0 \end{array} \right.$

$du = \sec^2 x dx$

$$\int_0^1 (u^2 + 1) u^2 du$$

9. $\int \frac{x}{(x-1)^2} dx$

a) $\ln|x-1| + \frac{1}{x-1} + C$

b) $\ln|x-1| - \frac{1}{x-1} + C$

c) $\ln|x-1| + \frac{1}{3(x-1)^2} + C$

d) $\ln|x-1| - \frac{1}{3(x-1)^2} + C$

e) $\ln|x-1| + \frac{3}{(x-1)^2} + C$

$$u = x-1 \rightarrow x = u+1$$

$$du = dx$$

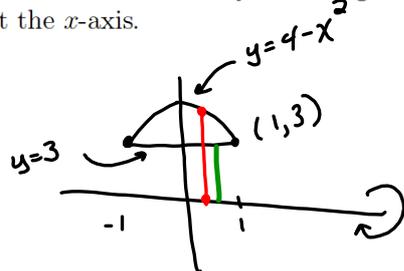
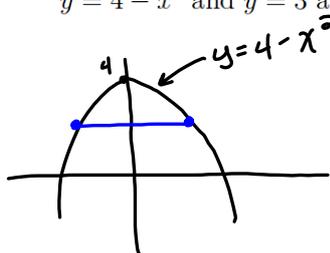
$$\int \frac{u+1}{u^2} du = \int \left(\frac{1}{u} + \frac{1}{u^2} \right) du$$

$$= \ln|u| - \frac{1}{u} + C$$

$$= \ln|x-1| - \frac{1}{x-1} + C$$

Part II - Work Out Problems

10. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the x -axis.



$$R = 4 - x^2$$

$$r = 3$$

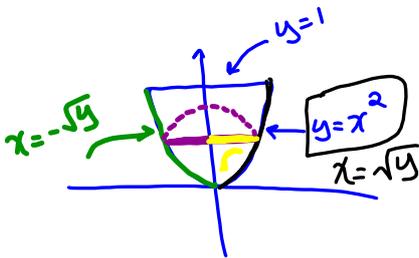
washers:
$$V = \int_{-1}^1 \pi (R^2 - r^2) dx$$

$$= 2 \int_0^1 \pi (R^2 - r^2) dx$$

$$= 2\pi \int_0^1 ((4 - x^2)^2 - 9) dx$$

Foil + integrate

11. The base of a solid is the region bounded by $y = x^2$ and $y = 1$. Cross-sections perpendicular the y-axis are semi-circles. Set up but do not evaluate an integral that gives the volume of the solid.



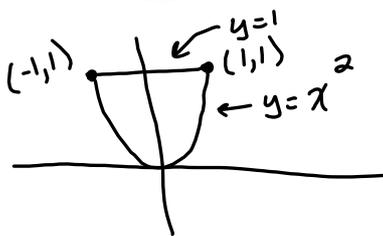
$$V = \int_0^1 (A_{\text{semicircle}}) dy$$

$$V = \int_0^1 \frac{\pi}{2} y dy$$

$$A_{\text{semicircle}} = \frac{1}{2} \pi r^2$$

$$r = \sqrt{y}$$

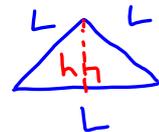
$$= \frac{1}{2} \pi y$$



same base, but now cross-sections perpendicular to the x axis are equilateral triangles.

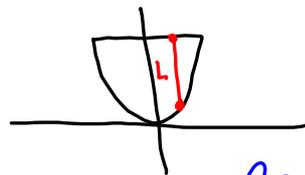
$$V = \int_{-1}^1 (A_{\text{equilateral triangle}}) dx$$

Fact: The area of an equilateral triangle with side length L is $A = \frac{\sqrt{3}}{4} L^2$



what is L ?

$$V = \int_{-1}^1 \frac{\sqrt{3}}{4} (1-x^2)^2 dx$$

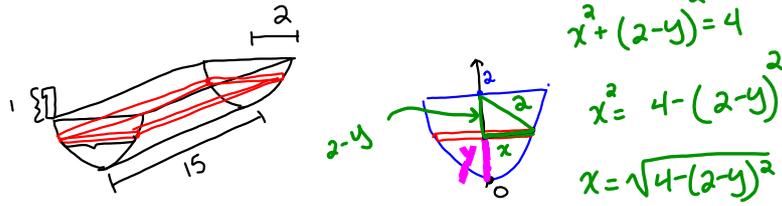


$$L = 1 - x^2$$

$$A = \frac{\sqrt{3}}{4} (1-x^2)^2$$

or $2 \int_0^1 \frac{\sqrt{3}}{4} (1-x^2)^2 dx$

12. A 15 m long trough with semicircular ends of radius 2 m is full of water. Set up but do not evaluate an integral that will compute the work required to pump all of the water out of a 1 m high spout. [Indicate on the picture where you are placing the axis and which direction is positive.] Note: The density of water is $\rho = 1000 \text{ kg/m}^3$ and the acceleration due to gravity is 9.8 m/s^2 .



$$x^2 + (2-y)^2 = 4$$

$$x^2 = 4 - (2-y)^2$$

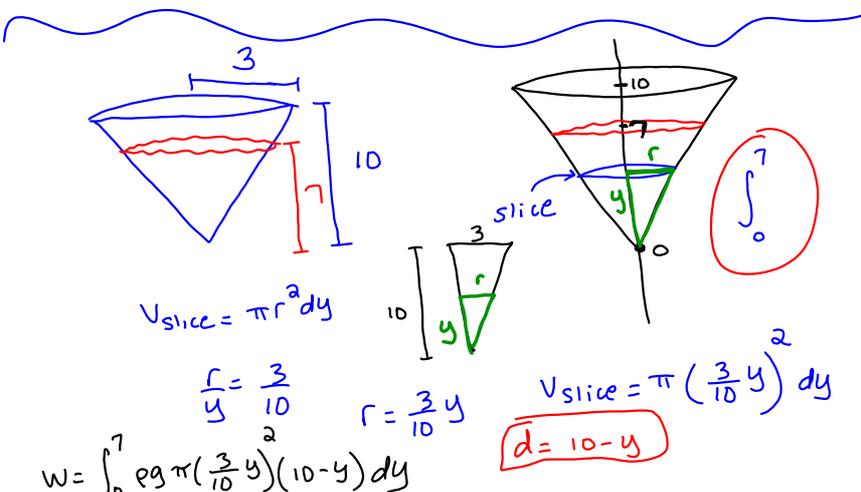
$$x = \sqrt{4 - (2-y)^2}$$

$$V_{\text{slice}} = (2x) 15 dy$$

$$= 2\sqrt{4 - (2-y)^2} 15 dy$$

$$d = 2 - y + 1 = 3 - y$$

$$W = \int_0^2 \underbrace{(9g) 2\sqrt{4 - (2-y)^2} 15}_{\text{Force}} \underbrace{(3-y)}_d dy$$



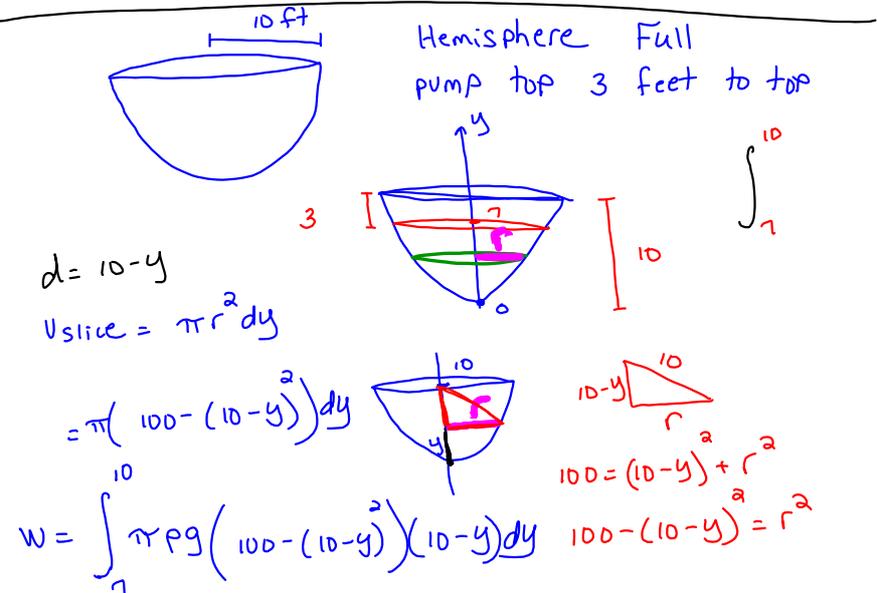
$$V_{\text{slice}} = \pi r^2 dy$$

$$\frac{r}{y} = \frac{3}{10} \implies r = \frac{3}{10}y$$

$$V_{\text{slice}} = \pi \left(\frac{3}{10}y\right)^2 dy$$

$$d = 10 - y$$

$$W = \int_0^7 9g \pi \left(\frac{3}{10}y\right)^2 (10 - y) dy$$



Hemisphere Full pump top 3 feet to top

$$d = 10 - y$$

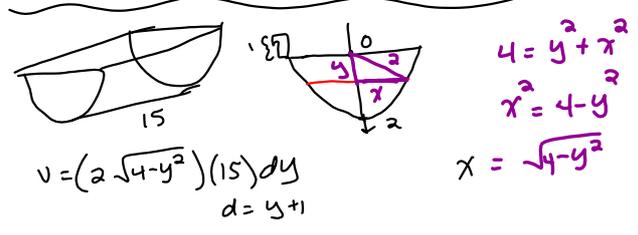
$$V_{\text{slice}} = \pi r^2 dy$$

$$= \pi (100 - (10-y)^2) dy$$

$$W = \int_7^{10} \pi 9g (100 - (10-y)^2) (10-y) dy$$

$$100 = (10-y)^2 + r^2$$

$$100 - (10-y)^2 = r^2$$



$$4 = y^2 + x^2$$

$$x^2 = 4 - y^2$$

$$x = \sqrt{4 - y^2}$$

$$V = (2\sqrt{4 - y^2})(15) dy$$

$$d = y + 1$$

13. Using cylindrical shells, set up but do not evaluate an integral that gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.

14. Consider the region R bounded by $y = \sqrt{x} + 3$, $y = 3$, $x = 16$

a.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the x -axis

b.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the y -axis

c.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $x = -1$

d.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $y = 10$.

15. Find $\int \sec^5 x \tan^3 x dx.$

$$= \int \underbrace{\sec^4 x}_{u^4} \underbrace{\tan^2 x}_{\sec^2 x - 1} \underbrace{\sec x \tan x}_{du} dx$$

$u = \sec x$
 $du = \sec x \tan x dx$

$$= \int u^4 (u^2 - 1) du$$

$$\int u^6 - u^4 du$$

16. Find $\int \sin^5(3x) \cos^2(3x) dx.$

17. Evaluate $\int \arccos x dx.$

$u = \arccos x$

$dv = dx$

u	dv
$\arccos x$	1
$-\frac{1}{\sqrt{1-x^2}}$	x

\int

$$x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx$$

$u = 1 - x^2$
 $du = -2x dx$

$$-\frac{1}{2} \int u^{-\frac{1}{2}} du$$

$$-\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = -\sqrt{u}$$

$$= -\sqrt{1-x^2}$$

$$x \arccos x - \sqrt{1-x^2} + C$$

17. Evaluate $\int e^x \cos(2x) dx$.