

Spring 2019 Math 152

courtesy: Amy Austin
(covering sections 7.3-11.2)

Section 7.3

1. $\int \frac{dx}{x^2\sqrt{x^2-1}}$

2. $\int x^3\sqrt{x^2+4} dx$

3. $\int_0^{2/3} \sqrt{4-9x^2} dx$

4. After performing the correct trigonometric substitution, rewrite the integral $\int_{-1}^1 \sqrt{x^2+2x+5} dx$ in terms of θ . Do not integrate.

Section 7.4

5. $\int_2^3 \frac{x^3+1}{x^2(x-1)} dx$

6. $\int \frac{x+1}{x^2-4} dx$

7. $\int \frac{2x^2-x+4}{x^3+4x} dx$

Section 7.8

8. $\int_e^\infty \frac{dx}{x(\ln x)^2}$

9. $\int_1^9 \frac{1}{\sqrt[3]{x-9}} dx$

10. $\int_{-1}^2 \frac{1}{x^4} dx$

11. Use the comparison theorem to determine whether the following improper integrals converge or diverge:

a.) $\int_1^\infty \frac{1}{x+e^{2x}} dx$

b.) $\int_5^\infty \frac{x}{x^{3/2}-x-1} dx$

c.) $\int_1^\infty \frac{\sin^4(x)+2}{x^3+1} dx$

d.) $\int_2^\infty \frac{\cos(4x)+7}{x+1} dx$

Section 11.1

12. Discuss the convergence or divergence of the following sequences:

a.) $a_n = \arcsin\left(\frac{-n+1}{2n+3}\right)$

b.) $a_n = \ln(3n+1) - \ln(4n^2)$

c.) $a_n = (-1)^n \frac{n}{n^2+1}$

d.) $a_n = (-1)^n \frac{n}{n+1}$

13. Which of the following sequences are both bounded and decreasing?

a.) $a_n = \frac{(-1)^n}{n^2}$

b.) $a_n = \frac{n}{(\ln n)^2}$

c.) $a_n = 1 + e^{-n}$

d.) $a_n = 2^{-n} \cos(n\pi)$

14. Given the recursive sequence is bounded and increasing, find the next 2 terms of the sequence and find value of the limit, if it exists.

$$a_1 = 2, a_{n+1} = 4 - \frac{3}{a_n}.$$

Section 11.2

15. What does the Test For Divergence say and which of the following series diverge using this test?

a.) $\sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)}$

b.) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

c.) $\sum_{n=1}^{\infty} \arctan(\ln n)$

16. If the n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is

$s_n = \frac{n+1}{n+4}$, find:

- a.) The sum of the first 200 terms.
 b.) The sum of the series.
 c.) a_n
17. Find the sum of the series. If the series diverges, explain why.

a.) $\sum_{n=1}^{\infty} \frac{n}{3(n+1)}$

b.) $\sum_{n=1}^{\infty} \left(\sin \frac{1}{n} - \sin \frac{1}{n+1} \right)$

c.) $\sum_{n=1}^{\infty} \frac{10}{n(n+2)}$

d.) $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+4}\right)$

e.) $\sum_{n=1}^{\infty} \ln\left(\frac{n+3}{n+4}\right)$

f.) $\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1}$

g.) $\sum_{n=0}^{\infty} (-3)^{n+1} \left(\frac{2}{5}\right)^n$

h.) $\sum_{n=0}^{\infty} \frac{2^{3n}}{(-5)^{n+1}}$

i.) $\sum_{n=0}^{\infty} \frac{(-1)^n + 3^n}{4^n}$

j.) $2 - \frac{4}{7} + \frac{8}{49} - \frac{16}{343} + \dots$

Section 11.3

15. Does $\sum_{n=0}^{\infty} n^2 e^{-n^3}$ converge or diverge? Support your answer.

16. Does $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converge or diverge? Support your answer.

17. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$. Using the Remainder Estimate for the Integral Test, find a value of n that will ensure the error in the approximation, s_n , is less than $\frac{1}{95}$. Express your answer as ' $n > \underline{\hspace{2cm}}$ '.

18. For the series $\sum_{n=1}^{\infty} \frac{5}{n^4}$, find s_6 , the sum of the first 6 terms. Using the Remainder Estimate for the Integral Test, estimate the error, R_6 , in using the sum of the first 6 terms as an approximation to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

19. Consider the series $\sum_{n=1}^{\infty} \frac{3}{n^4}$. Using the Remainder Estimate for the Integral Test, find a value of n that will ensure the error in the approximation, s_n , is less than $\frac{1}{95}$. Express your answer as ' $n > \underline{\hspace{2cm}}$ '.