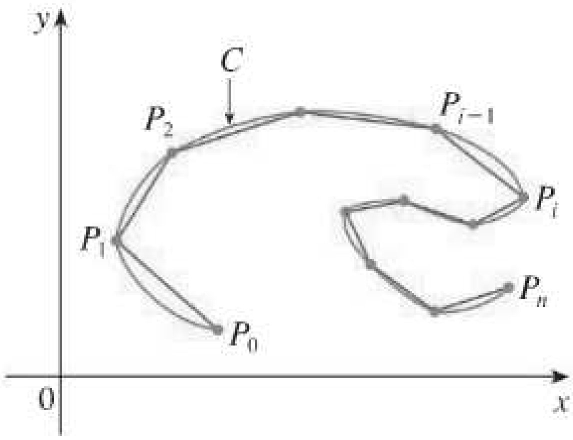
Section 10.2: Calculus with Parametric Curves

Calculus with Parametric Curves

Arclength If $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, then the length of the curve from $t = \alpha$ to $t = \beta$ is

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$



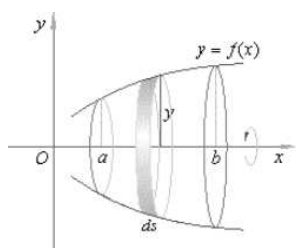
1. Find the length of the curve $x = 3t - t^3$, $y = 3t^2$, $0 \leq t \leq 2$.

2. Find the length of the curve $x = 2t^2 + \frac{1}{t}$, $y = 8\sqrt{t}$, $1 \leq t \leq 3$

Surface Area

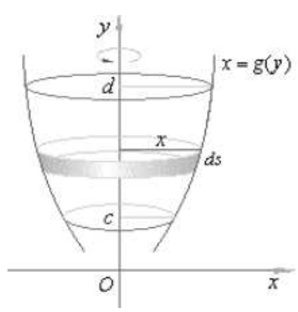
• If the curve $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, is revolved around the x axis, then the resulting surface area is given by

$$SA = 2\pi \int_{\alpha}^{\beta} g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$



• If the curve $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, is revolved around the y axis, then the resulting surface area is given by

$$SA = 2\pi \int_{\alpha}^{\beta} f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$



3. Find the surface area obtained by revolving the curve $x = 3t - t^3$, $y = 3t^2$, $0 \le t \le 1$ about the x-axis.

4. Find the surface area obtained by revolving the curve $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 1$ about the y-axis.