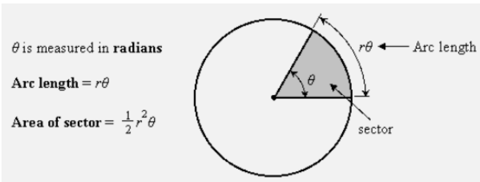
Section 10.4 Areas and Lengths in Polar Coordinates

In this section, we develop a formula for the area of a region whose boundary is given by a polar equation. Recall the area of a sector is $A = \frac{1}{2}r^2\theta$, where r is the radius and θ is the radian measure of the central angle.



Let R be the region, illustrated the figure below, bounded by the polar curve $r = f(\theta)$ and the rays $\theta = a$ and $\theta = b$.

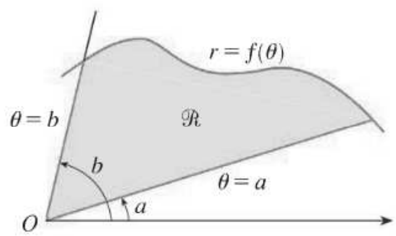


FIGURE 2

The area A of the polar region R is $A = \int_a^b \frac{1}{2}\left[f(\theta)\right]^2 d\theta = \int_a^b \frac{1}{2}r^2\, d\theta$.

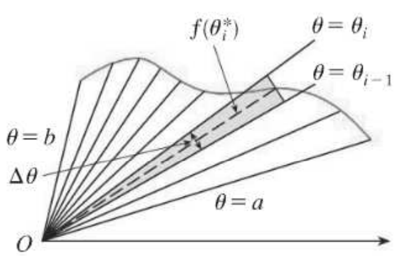
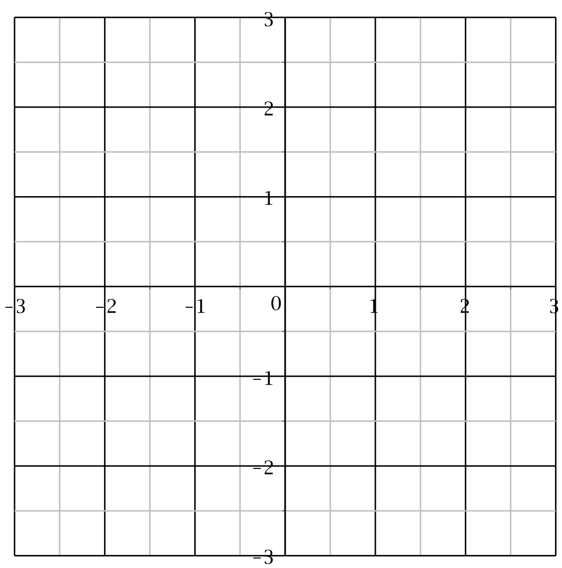


FIGURE 3

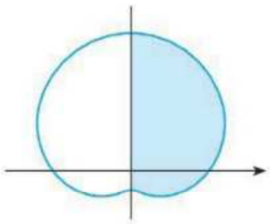
1. Find the area of the region that is bounded by $r = \frac{1}{\theta}$ that lies in the sector $\frac{\pi}{2} \le \theta \le 2\pi$.

2. Find the area of the region that is bounded by $r = \sin\theta + \cos\theta$ that lies in the sector $0 \leq \theta \leq \pi$.

3. Sketch the curve $r = 2\sin\theta$ and find the area it encloses.

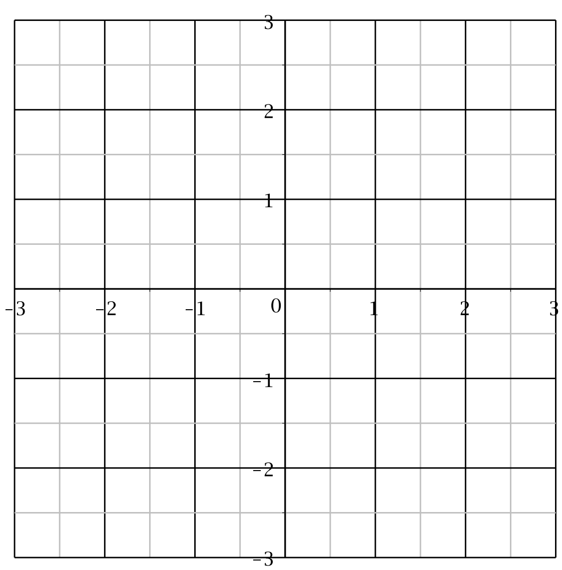


4. Find the area of the shaded region.



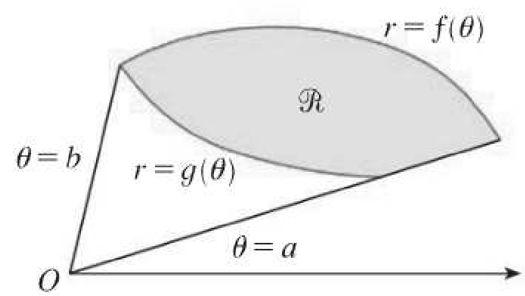
$r = 4 + 3\sin\theta$

5. Sketch the curve $r = 1 - \sin\theta$ and find the area it encloses.

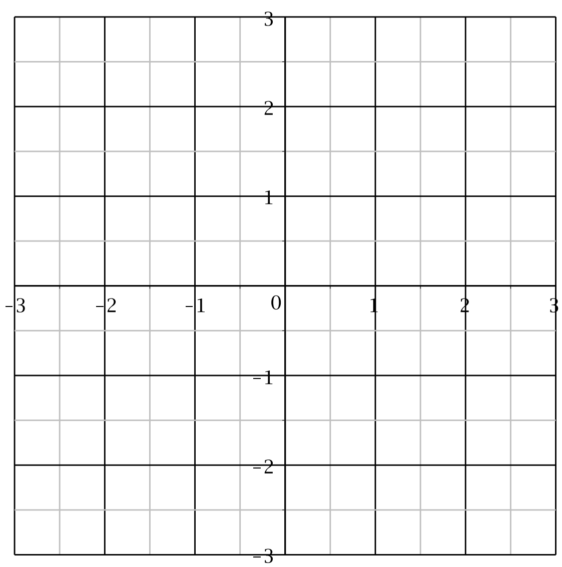


Let R be the region that is bounded by $f(\theta)$ and $g(\theta)$, $a \le \theta \le b$, as shown below. Then the area of R is

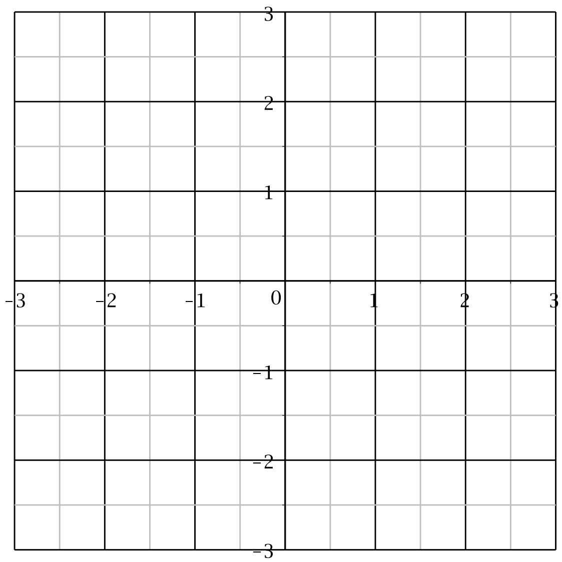
$$A = \int_a^b \frac{1}{2}\left([f(\theta)]^2 - [g(\theta)]^2\right)\, d\theta$$



6. Sketch the region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$. Find the area of the region.



7. Sketch the region inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$. Find the area of the region.



8. Set up but do not evaluate an integral that gives the area of one loop of the polar curve $r = \sin(3\theta)$.

The lenth of the polar curve $r = f(\theta)$ $a \leq \theta \leq b$, is $L = \displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$.

9. Find the lenth of the polar curve $r = \theta^2$, $0 \leq \theta \leq \dfrac{\pi}{2}$

10. Set up but do not evaluate an integral that gives the length of one loop of the curve $r = \cos(2\theta)$.