Section 11.4 The Comparison Tests

The Comparison Test: Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{i=1}^{\infty} b_n$ are series of **positive terms** (Note: series need not start at 1.)

• If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also convergent. In other words, "if the larger series converges, so does the smaller series".

• If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also divergent. In other words, "if the smaller series diverges, so does the larger series".

Note: if the larger series diverges, no conclusion can be made about the smaller series. Likewise, if the smaller series converges no conclusion can be made about the larger series.

1. Determine whether the following series converge or diverge. Fully support your answer.

a.)
$$\sum_{n=1}^{\infty} \frac{5n^4}{n^4 + n^2 + 1}$$

b.)
$$\sum_{n=1}^{\infty} \frac{n^4}{n^8 + n^2 + 1}$$

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c.)
$$\sum_{n=3}^{\infty} \frac{n^2 + 1}{\sqrt{n^5 - n}}$$

d.)
$$\sum_{n=1}^{\infty} \frac{\sin n + 5}{n^3 \sqrt{n}}$$

e.)
$$\sum_{n=1}^{\infty} \frac{5 + \sin n}{\sqrt{n}}$$

f.)
$$\sum_{n=1}^{\infty} \frac{5 + \arctan(n)}{n^4}$$

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g.)
$$\sum_{n=1}^{\infty} \frac{3}{5^n + n}$$

h.)
$$\sum_{n=5}^{\infty} \frac{n\sqrt{n}}{8n^2 + 6n}$$

The Limit Comparison Test: If the Comparison Test is inconclusive, then we may apply the Limit Comparison Test, which determines whether two series behave similarly for vert large values of n.

Suppose
$$\sum_{n=1}^{\infty} a_n$$
 and $\sum_{n=1}^{\infty} b_n$ are series of positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ and finite, then either both series converge or both diverge.

Note: If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$ or $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$, then the test fails and therefore we need to apply another test.

2. Determine whether the following series converge or diverge.

a.)
$$\sum_{n=5}^{\infty} \frac{n\sqrt{n}}{8n^2 + 6n}$$

b.)
$$\sum_{n=1}^{\infty} \frac{n^4 - n^3 + 1}{\sqrt{n^{10} - n^6 + 3}}$$

c.)
$$\sum_{n=1}^{\infty} \frac{n+5}{(2n+1)^3+2n}$$

d.)
$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$