Section 11.5 Alternating Series

The convergence tests we have discussed so far apply only to series of positive terms. In this section and the next we learn how to deal with series that are not necessarily positive. Of particular importance are *alternating series*, whose terms alternate signs.

Definition: An alternating series is a series whose terms alternate signs. For example,

 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is an alternating series. We would like to know under what conditions does an alternating series converge?

The Alternating Series Test: The alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$, where $a_n > 0$, converges if it satisfies both conditions given below:

- $a_{n+1} \leq a_n$ for all n (if the sequence $\{a_n\}$ is decreasing).
- $\lim_{n \to \infty} a_n = 0$

Illustration as to why this is true.

Consider $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, where $b_n > 0$ and a_n decreases to zero.

 $s_1 = b_1$

- $s_2 = b_1 b_2$
- $s_3 = b_1 b_2 + b_3$
- $s_4 = b_1 b_2 + b_3 b_4$

$$s_5 = b_1 - b_2 + b_3 - b_4 + b_5$$



1. Determine whether the following series are convergent.

a.)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+3}}$$

b.)
$$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \dots$$

c.)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$$

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d.)
$$\sum_{n=1}^{\infty} (-1)^{n-1} 2^{-n}$$

e.)
$$\sum_{n=1}^{\infty} (-1)^n 2^{3/n}$$

f.)
$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$$

Remainder Estimate and The Alternating Series Estimation Theorem

If $\sum_{n=1}^{\infty} (-1)^n a_n$, $a_n > 0$, is a convergent alternating series, and a partial sum $s_n = \sum_{i=1}^n (-1)^i a_i$ is used to approximate the sum of the series with remainder R_n , then

$$|R_n| \le a_{n+1}$$

2. Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

a.) Prove the series is convergent.

b.) Use s_6 to approximate the sum of the series and use the Alternating Series Estimation Theorem to estimate the error in using the 6th partial sum to approximate the sum of the series.

c.) Determine the minimum number of terms we need to add in order to find the sum with error less than $\frac{1}{150}$.

3. approximate
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$
 to within an accuracy of 10^{-2} .