Section 11.6 Absolute Convergence and the Ratio Test

Given any series, $\sum_{n=1}^{\infty} a_n$, we can consider the corresponding series $\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + |a_3| + ...$

We learned in section 11.5 that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ both converge by the alternating series test. What happens if we consider:

a.) $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n}\right|$

b.) $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right|$

Definition: A series is **absolutely convergent** if $\sum_{n=1}^{\infty} |a_n|$ converges. If $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ diverges, then the $\sum_{n=1}^{\infty} a_n$ is **conditionally convergent**. Note: If $\sum_{n=1}^{\infty} a_n$ is a series of positive terms, and $\sum_{n=1}^{\infty} a_n$ converges, then by default it is absolutely convergent.

If a series is absolutely convergent, then it is convergent.

1. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

   a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

   b.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

   c.) $\sum_{n=1}^{\infty} \frac{\cos n}{n^4}$

The Ratio Test:

- If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).
- If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the test fails.

2. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

a.) $\sum_{n=1}^{\infty} \frac{3^n}{n^3(-2)^n}$

b.) $\sum_{n=1}^{\infty} \frac{n^{10}(-100)^n}{n!}$

c.) $\sum_{n=1}^{\infty} \frac{(2n+1)!}{n!}$

3. For which of the following series is the ratio test inconclusive?

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

b.) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

c.) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(n+1)!}$