Section 11.8 Power Series

Definition: A **Power Series** is a series of the form $\sum_{n=1}^{\infty} c_n(x-a)^n$, where x is the variable and the c_n's are called the coefficients of the series. More generally, $\sum_{n=1}^{\infty} c_n(x-a)^n$ is called a power series *centered* at $x = a$, or a power series *about* a. Further, the interval of convergence is the set of all x for which the power series converges, denoted by I. We define the the radius of convergence to be $R = \frac{1}{2} \cdot$ (length of I).

Theorem: For a given power series $\sum_{n=1}^{\infty} c_n(x-a)^n$ there are only three possibilities:

(i) The series converges only for $x = a$. In this case, $R = 0$ and $I = \{a\}$.

(ii) The series converges for all x. In this case, $R = \infty$ and $I = (-\infty, \infty)$.

(iii) There is a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$. In this case, R the radius of convergence and the interval of convergence is AT LEAST $I = (-R + a, R + a)$. We must test the endpoints for convergence.

1. For the following power series, find the radius and interval of convergence.

   a.) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n^2 + 1}$

b.) $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{4n-1}$

c.) $\sum_{n=0}^{\infty} (2n)!(2x-5)^n$

d.) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(2n+1)!}$

e.) $\sum_{n=2}^{\infty} \frac{(-1)^n(x+1)^n}{2^n \ln n}$

2. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series:

   a.) $\sum_{n=0}^{\infty} c_n (2)^n$

   b.) $\sum_{n=0}^{\infty} c_n (8)^n$

   c.) $\sum_{n=0}^{\infty} c_n (4)^n$

   d.) $\sum_{n=0}^{\infty} c_n (-5)^n$

3. Suppose it is known that $\sum_{n=0}^{\infty} c_n (x-1)^n$ converges when $x = 3$. On what interval are we certain the series converges?