Section 5.5: Integration by Substitution

The Substitution Rule: If $u = g(x)$ is a differentiable function, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

Note: Typically, u is chosen so that du is a factor of the integrand.

1. $\int 2t^2(t^3-1)^3\,dt$

2. $\int \frac{\sec^2 x}{\sqrt{\tan x + 9}}\,dx$

3. $\displaystyle\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}}\,dx$

4. $\displaystyle\int \frac{x^2}{(1-x)^4}\,dx$

5. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

6. $\int_0^{\pi/4} e^{\sin(2t)} \cos(2t)\, dt$

7. $\int \tan(x)\, dx$

8. $\int \frac{\arctan x}{1+x^2}\, dx$

9. $\displaystyle\int \frac{x+1}{x^2+1}\,dx$

10. $\displaystyle\int_{e^3}^{e^4} \frac{1}{x\ln x}\,dx$

Note: If the choice of u is linear (degree 1), then du is a constant multiple of dx and hence can be divided out of the integral. To illustrate, let's find $\displaystyle\int e^{kx}\,dx$.

11. $\int \left(\cos(5x) + e^{-10x} - \dfrac{3}{4x-1}\right) dx$

12. $\int (\sin(3\alpha) - \sin(3x))\, dx$

13. $\int \dfrac{\sin x}{\sqrt{1-\cos^2 x}}\, dx$

14. $\int \dfrac{x}{1+x^4}\, dx$