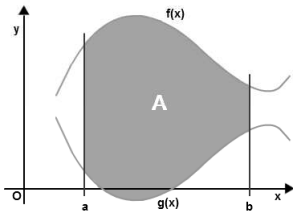
Section 6.1: Area Between Curves

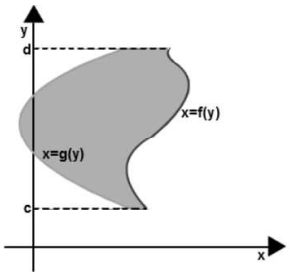
The area A bounded by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$ and $x = b$, where $f(x) \geq g(x)$ for all x in the interval $[a, b]$ is

$$A = \int_a^b (f(x) - g(x))\, dx$$

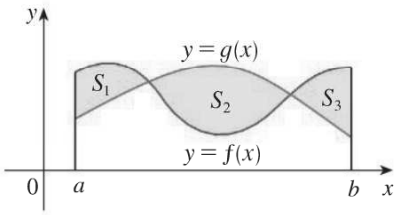


The area A bounded by the curves $x = f(y)$, $x = g(y)$ and the lines $y = c$ and $y = d$, where $f(y) \geq g(y)$ for all y in the interval $[c, d]$ is

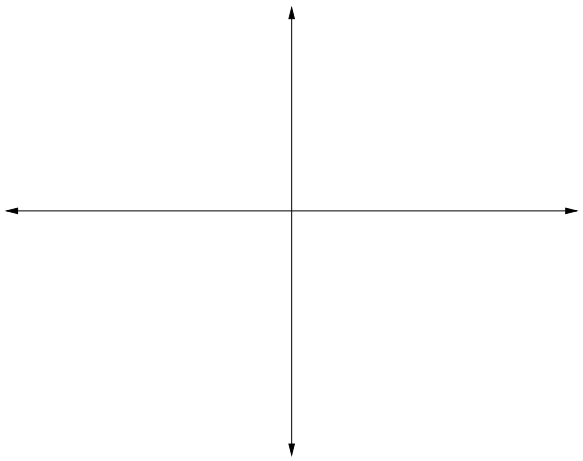
$$A = \int_c^d (f(y) - g(y))\, dy$$



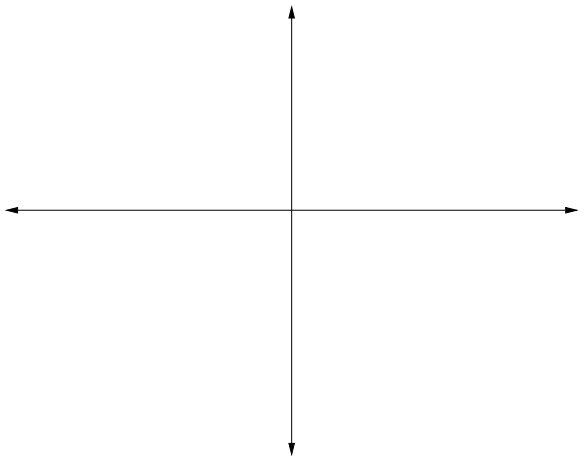
If we are asked to find the area bounded by the curves $y = f(x)$, $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x, we must split the integral at each intersection point.



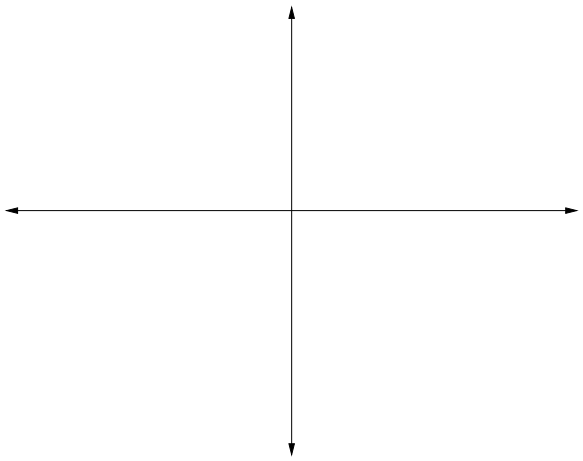
1. Sketch the region R bounded by $y = \sin x$, $y = 0$, $x = 0$, $x = \dfrac{\pi}{3}$. Find the area of R.



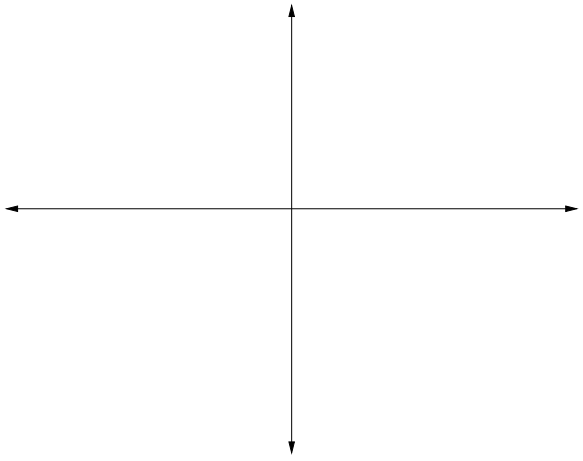
2. Sketch the region R bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \dfrac{2\pi}{3}$. Find the area of R.



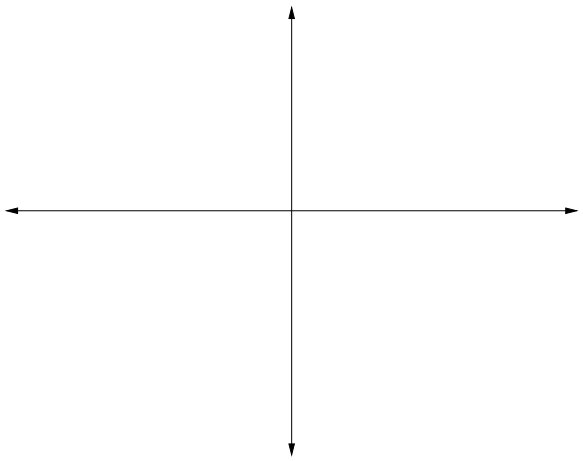
3. Sketch the region R bounded by $y = e^x$, $y = x^2 - 1$, $-1 \leq x \leq 1$. Find the area of R.



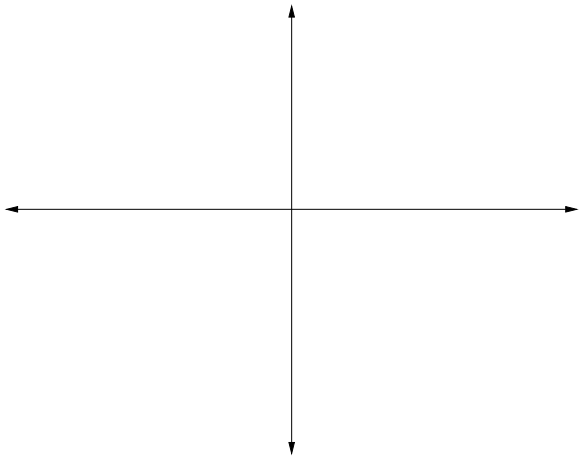
4. Sketch the region R bounded by $y = 4x^2$, $y = x^2 + 3$. Find the area of R.



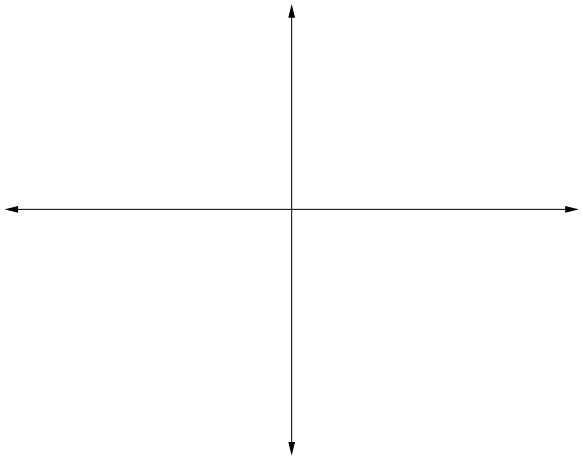
5. Sketch the region R bounded by $y = x^2 + 1$, $y = 3 - x^2$, $x = -2$, $x = 2$. Set up but do not evaluate an integral that gives the area of R.



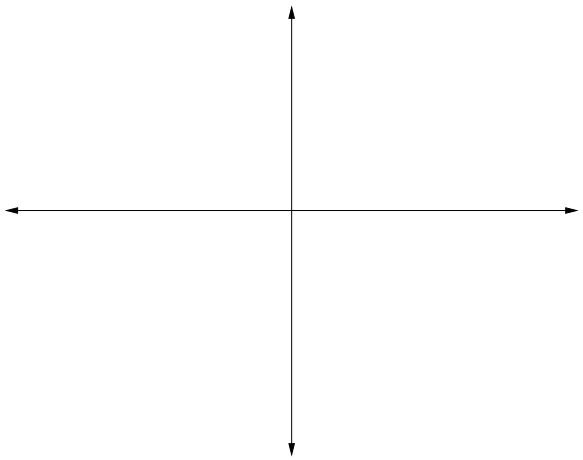
6. Sketch the region R bounded by $y = |x|$, $y = x^2 - 2$. Set up but do not evaluate an integral that gives the area of R.



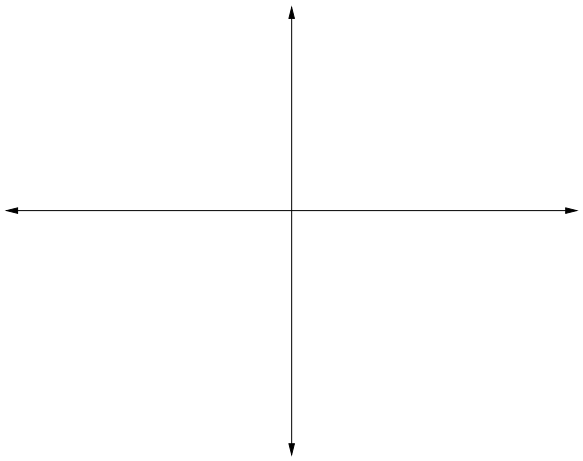
7. Sketch the region R bounded by $y = \sin x$, $y = \cos x$, $x = -\dfrac{\pi}{2}$, $x = \dfrac{\pi}{2}$. Set up but do not evaluate an integral that gives the area of R.



8. Sketch the region R bounded by $x = 2 - 2y^2$, $x = 2y^2 - 2$. Find the area of R.



9. Sketch the region R bounded by $y^2 = x$, $x - 2y = 3$. Find the area of R.



10. Sketch the region R bounded by $y = x^2$, the tangent line to this parabola at $(1, 1)$ and the x-axis. Find the area of R.