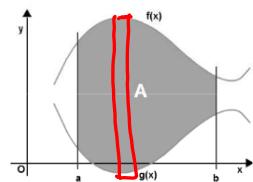


Section 6.1: Area Between Curves

The area A bounded by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$ and $x = b$, where $f(x) \geq g(x)$ for all x in the interval $[a, b]$ is

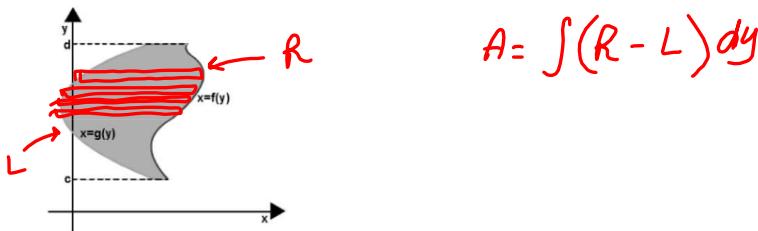
$$A = \int_a^b (f(x) - g(x)) dx$$



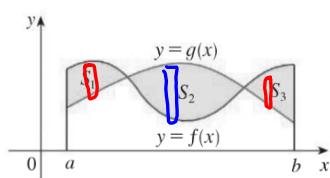
$$A = \int_a^b (f(x) - g(x)) dx$$

The area A bounded by the curves $x = f(y)$, $x = g(y)$ and the lines $y = c$ and $y = d$, where $f(y) \geq g(y)$ for all y in the interval $[c, d]$ is

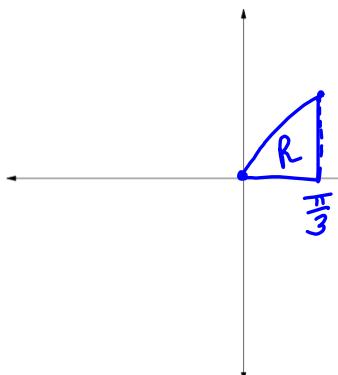
$$A = \int_c^d (f(y) - g(y)) dy$$



If we are asked to find the area bounded by the curves $y = f(x)$, $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x , we must split the integral at each intersection point.



1. Sketch the region R bounded by $y = \sin x$, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$. Find the area of R .



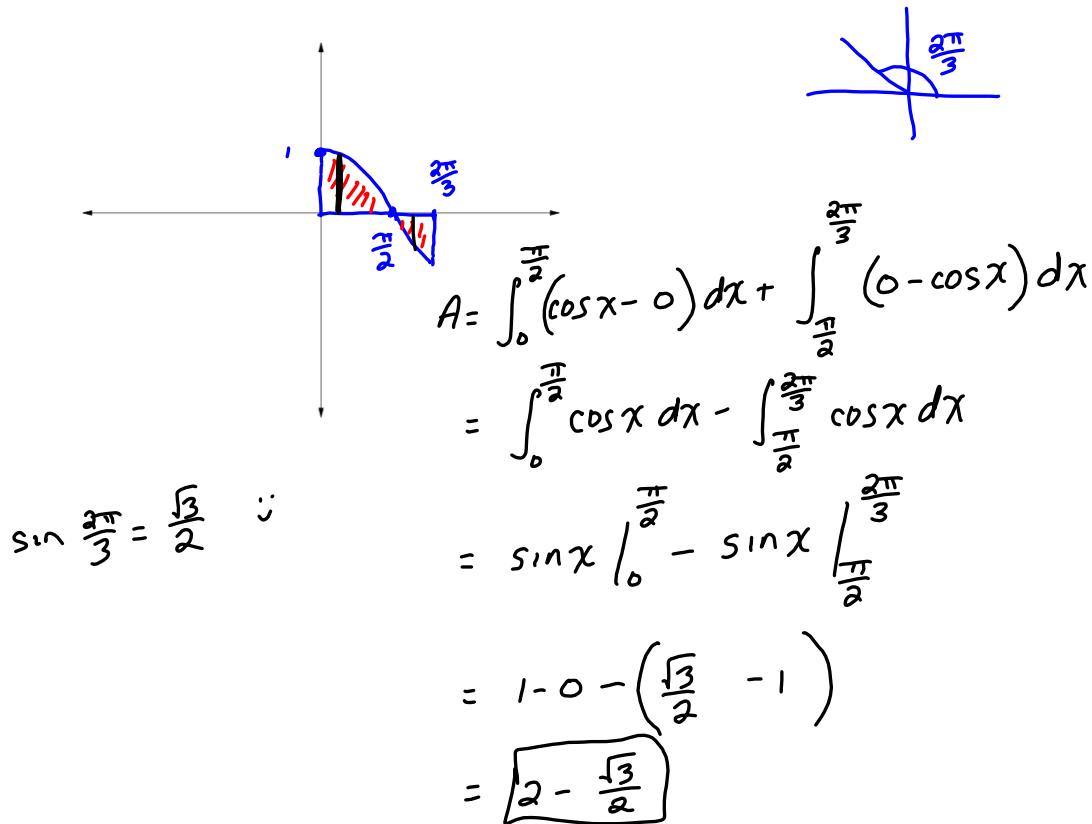
$$A = \int_0^{\frac{\pi}{3}} (\sin x - 0) dx$$

$$A = -\cos x \Big|_0^{\frac{\pi}{3}}$$

$$A = -\frac{1}{2} - (-1)$$

$$\boxed{A = \frac{1}{2}}$$

2. Sketch the region R bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{2\pi}{3}$. Find the area of R .



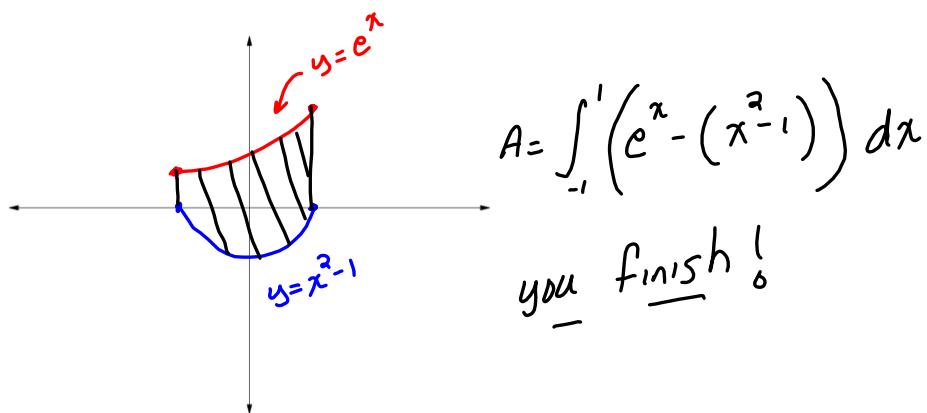
$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$= \sin x \Big|_0^{\frac{\pi}{2}} - \sin x \Big|_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$$

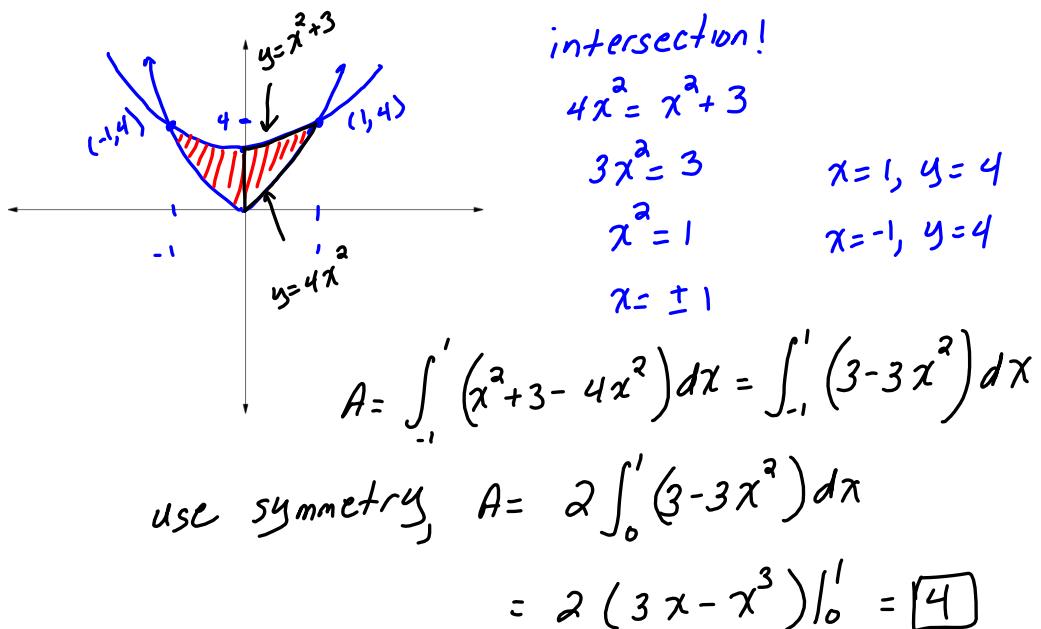
$$= 1 - 0 - \left(\frac{\sqrt{3}}{2} - 1 \right)$$

$$\boxed{2 - \frac{\sqrt{3}}{2}}$$

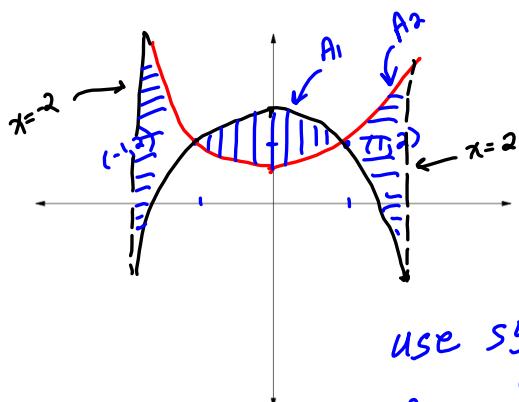
3. Sketch the region R bounded by $y = e^x$, $y = x^2 - 1$, $-1 \leq x \leq 1$. Find the area of R .



4. Sketch the region R bounded by $y = 4x^2$, $y = x^2 + 3$. Find the area of R .



5. Sketch the region R bounded by $y = x^2 + 1$, $y = 3 - x^2$, $x = -2$, $x = 2$. Set up but do not evaluate an integral that gives the area of R .



$$\text{intersect: } x^2 + 1 = 3 - x^2$$

$$2x^2 = 2$$

$$x = \pm 1$$

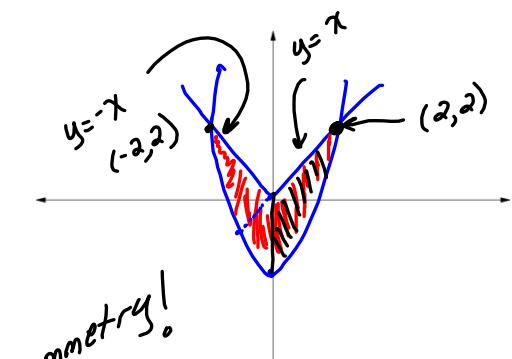
$$(1, 2), (-1, 2)$$

use symmetry!

$$A = 2 \int [A_1 + A_2]$$

$$= 2 \left[\int_{-1}^1 (3 - x^2 - (x^2 + 1)) dx + \int_1^2 (x^2 + 1 - (3 - x^2)) dx \right]$$

6. Sketch the region R bounded by $y = |x|$, $y = x^2 - 2$. Set up but do not evaluate an integral that gives the area of R .



$$\text{if } x > 0, |x| = x$$

$$x = x^2 - 2$$

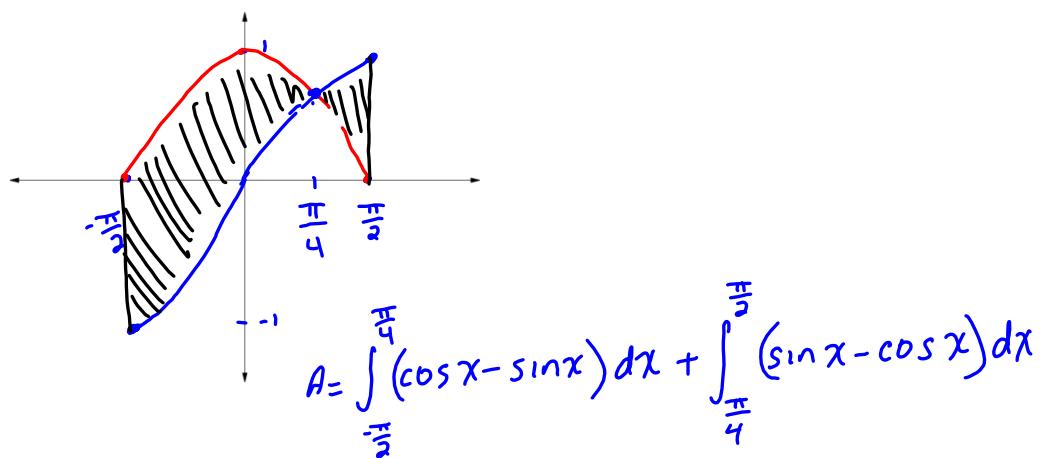
$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, x = -1$$

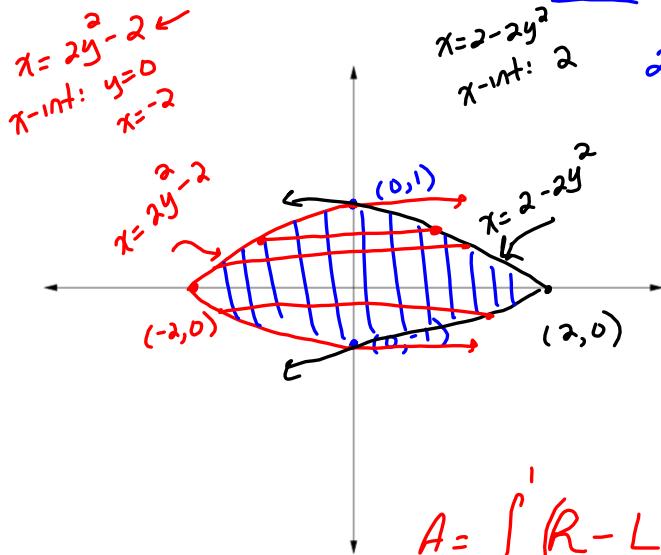
$$A = 2 \int_0^2 (x - (x^2 - 2)) dx$$

7. Sketch the region R bounded by $y = \sin x$, $y = \cos x$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$. Set up but do not evaluate an integral that gives the area of R .



Add this problem

- Sketch the region R bounded by $x = 2 - 2y^2$, $x = 2y^2 - 2$. Find the area of R .



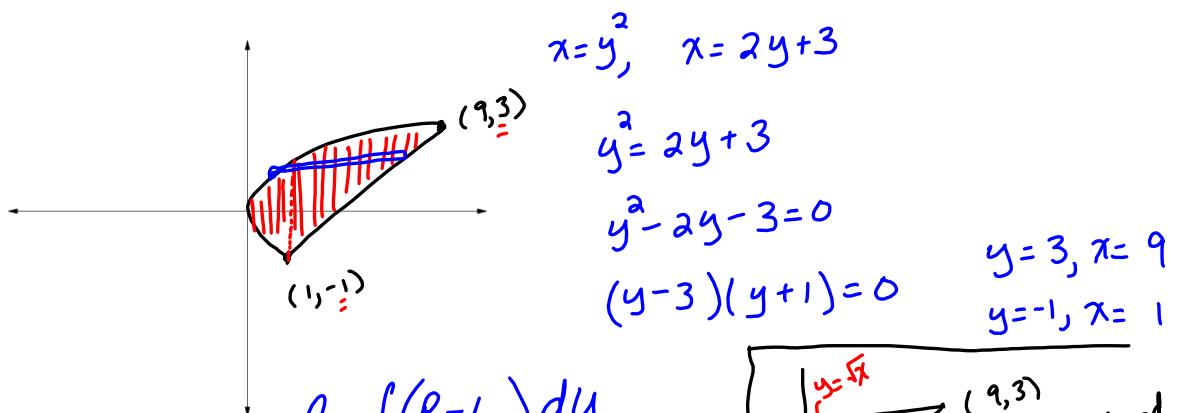
$$A = \int_{-1}^1 (R - L) dy \text{ or symmetry } 2 \int_0^1 R - L dy$$

$$2 \int_0^1 (2 - 2y^2 - (2y^2 - 2)) dy$$

$$2 \int_0^1 (4 - 4y^2) dy = 2 \left(4y - \frac{4}{3} y^3 \right) \Big|_0^1$$

$$= 2 \left(4 - \frac{4}{3} \right) = 2 \left(\frac{8}{3} \right)$$

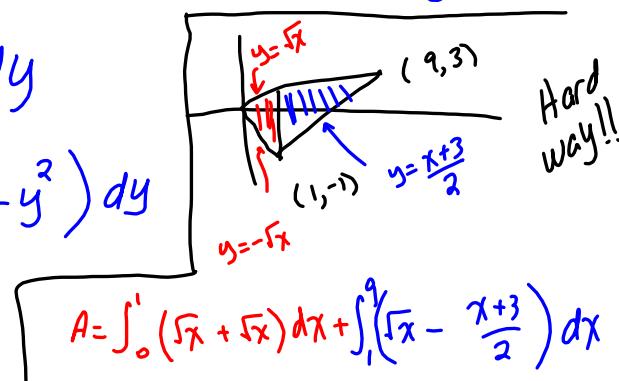
8. Sketch the region R bounded by $y^2 = x$, $x - 2y = 3$. Find the area of R .



$$A = \int (R - L) dy$$

$$= \int_{-1}^3 (2y + 3 - y^2) dy$$

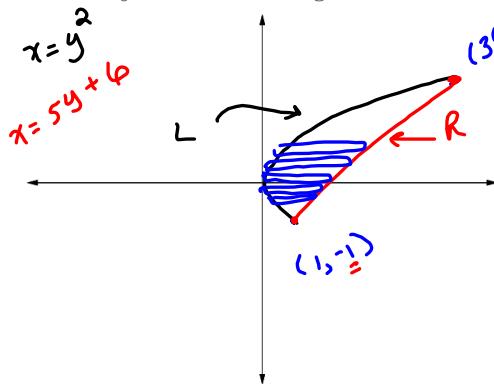
$$A = \int_0^1 (\sqrt{x} + \sqrt{x}) dx + \int_1^9 (\sqrt{x} - \frac{x+3}{2}) dx$$



Section 6.1 continued

Section 6.1: Warm Up

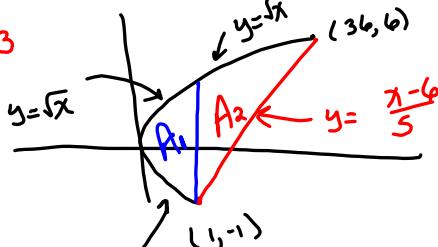
1. Sketch the region R bounded by $x = y^2$ and $x = 5y + 6$. Set up but do not evaluate an integral in terms of y and then an integral in terms of x that gives the area of this region.



$$\begin{aligned} & \textcircled{1} \text{ integral in terms of } y: \\ & y^2 = 5y + 6 \\ & y^2 - 5y - 6 = 0 \\ & (y-6)(y+1) = 0 \\ & y=6 \rightarrow x=36 \quad A = \int_{-1}^{6} (5y+6 - y^2) dy \\ & y=-1 \rightarrow x=1 \end{aligned}$$

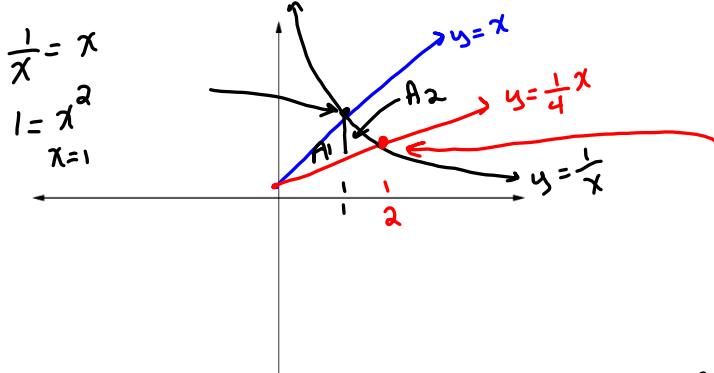
$\textcircled{2}$ integral in terms of x : T-B

$$\begin{aligned} & x = y^2, \quad x = 5y + 6 \\ & y = \pm\sqrt{x}, \quad y = \frac{x-6}{5} \end{aligned}$$



$$\begin{aligned} A &= A_1 + A_2 \\ &= \int_0^1 \underbrace{\sqrt{x} - (-\sqrt{x})}_{2\sqrt{x}} dx + \int_1^{36} \left(\sqrt{x} - \frac{x-6}{5} \right) dx \end{aligned}$$

2. Sketch the region R bounded by $y = \frac{1}{x}$, $y = x$, $y = \frac{1}{4}x$, $x \geq 0$. Set up but do not evaluate an integral that gives the area of R .

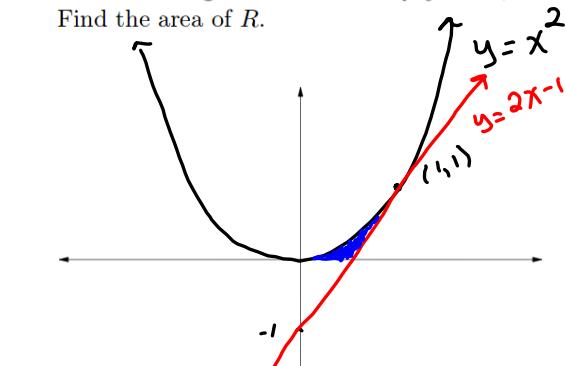


$$y = x, \quad y = \frac{1}{4}x$$

$$\begin{aligned} y &= \frac{1}{x} \\ \frac{1}{4}x &= \frac{1}{x} \\ x^2 &= 4 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} & \text{integral in terms of } x \\ A &= A_1 + A_2 = \int_0^1 \left(x - \frac{1}{4}x \right) dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x \right) dx \end{aligned}$$

9. Sketch the region R bounded by $y = x^2$, the tangent line to this parabola at $(1, 1)$ and the x -axis.
Find the area of R .



Recall: the slope of the tangent line to $f(x)$ at $x=a$ is $m = f'(a)$

$$f(x) = x^2, \quad a=1$$

$$m = f'(1) \quad f'(x) = 2x$$

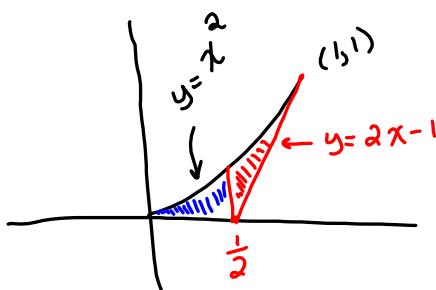
$$m = 2$$

$$\boxed{\text{point } (1, 1)}$$

tangent line

$$y-1 = 2(x-1)$$

$$\boxed{y = 2x - 1} \leftarrow$$



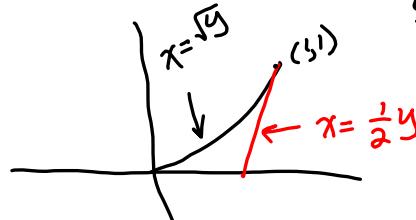
① integral in terms of x .

$$y = x^2, \quad y = 2x - 1$$

$$A = \int_0^1 (x^2 - 0) dx + \int_{\frac{1}{2}}^1 x^2 - (2x - 1) dx$$

② integral in terms of y

$$y = x^2 \rightarrow x = \sqrt{y}, \quad y = 2x - 1 \rightarrow x = \frac{y+1}{2}$$



$$\begin{aligned} A &= \int_0^1 \left(\frac{1}{2}y + \frac{1}{2} - y^{\frac{1}{2}} \right) dy \\ &= \frac{1}{2} \frac{y^2}{2} + \frac{1}{2}y - \frac{2}{3} y^{\frac{3}{2}} \Big|_0^1 \\ &= \frac{1}{4} + \frac{1}{2} - \frac{2}{3} = \boxed{\frac{1}{12}} \end{aligned}$$

