Section 7.2: Trigonometric Integrals

Identities needed in this section:

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $\cot^2 x + 1 = \csc^2 x$
4. $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$
5. $\cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$

Type I: Integrals of the form $\int \sin^m x \cos^n x \, dx$, where m and n are non-negative integers.

Case 1: *EITHER* m or n (or both) is odd. If the power on $\sin x$ is odd, factor out one $\sin x$ and let $u = \cos x$. If the power on $\cos x$ is odd, factor out one $\cos x$ and let $u = \sin x$. If both powers are odd, do one of the above mentioned but not both.

1. $\int \sin^4 x \cos^3 x \, dx$

2. $\int \sin^3(10x) \, dx$

3. $\displaystyle\int \cos^5 x\sqrt{\sin x}\,dx$

4. $\displaystyle\int_0^{\pi/2} \cos^3 x \sin^3 x\,dx$

Case 2: *BOTH* m and n are even. Use the identities $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$.

1. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\,dx$

2. $\displaystyle\int \frac{\cos^2(\ln x)}{x}\,dx$

Type II: Integrals of the form $\displaystyle\int \sec^m x \tan^n x\,dx$, where m and n are non-negative integers.

Case 1: The power on $\tan x$ is odd. Factor out $\sec x \tan x$ and let $u = \sec x$.

1. $\displaystyle\int \tan^3 x \sec^3 x\,dx$

2. $\displaystyle\int \cot^5 x \csc x\,dx$

Case 2: The power on $\sec x$ is even. Factor out $\sec^2 x$ and let $u = \tan x$.

1. $\displaystyle\int x\tan^4(x^2)\sec^4(x^2)\,dx$

2. $\displaystyle\int \tan^9 x\sec^4 x\,dx$. Note here, the power on tangent is odd *and* the power on secant is even. Hence treat it as a case 1 *OR* case 2.

3. $\displaystyle\int \tan^2 x\,dx$

Connecting volume of revolution with our new techniqes of integration.

1. Find the volume obtained by rotating the region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{2}$ about the x axis.

2. Find the volume obtained by rotating the region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{2}$ about the line $x = \dfrac{\pi}{2}$

3. Find the volume obtained by rotating the region bounded by $y = \sec^2 x$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{4}$ about the x axis.