Section 7.3: Trigonometric Substitution

Identities commonly used in this section:

a.) $\sin^2\theta + \cos^2\theta = 1$

b.) $\tan^2\theta = \sec^2\theta - 1$

c.) $\sin(2x) = 2\sin x\cos x$

A few interals you must know:

1. $\displaystyle\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$

   Proof: $\displaystyle\int \sec\theta\, d\theta = \int \sec\theta\left(\frac{\sec(\theta)+\tan(\theta)}{\sec(\theta)+\tan(\theta)}\right) d\theta = \int \left(\frac{\sec^2(\theta)+\sec(\theta)\tan(\theta)}{\sec(\theta)+\tan(\theta)}\right) d\theta.$

   If we let $u = \sec(\theta)+\tan(\theta)$, then $du = (\sec^2(\theta)+\sec(\theta)\tan(\theta))\, d\theta$, which gives

   $\displaystyle\int \frac{1}{u}\, du = \ln|u| = \ln|\sec(\theta)+\tan(\theta)|.$

2. $\displaystyle\int \sec^3\theta\, d\theta = \frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right) + C$

The general idea of integration by trigonometric substitution is to transform an algebraic integral that involves one of the general forms in the table below into a trig integral that can be integrated using the techniques of section 7.2.

Form	**Substitution**	Identity Used	Domain
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$\cos^2\theta = 1-\sin^2\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$\sec^2\theta = \tan^2\theta + 1$	$0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\tan^2\theta = \sec^2\theta - 1$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

1. $\int \frac{dx}{x^2\sqrt{1-x^2}}$

2. $\displaystyle\int \frac{\sqrt{x^2+9}}{x^4}\,dx$

3. $\displaystyle\int \frac{\sqrt{9x^2-4}}{x}\,dx$

4. $\displaystyle\int \frac{x^2}{\sqrt{1-9x^2}}\,dx$

5. $\displaystyle\int_{5\sqrt{2}}^{10} \frac{dx}{x^3\sqrt{x^2-25}}$

6. $\displaystyle\int_0^{2/3} \frac{1}{(4+9x^2)^{5/2}}\,dx$

7. $\displaystyle\int \frac{dx}{\sqrt{x^2 - 8x}}$

8. $\displaystyle\int \frac{1}{(x^2+6x+13)^{3/2}}\,dx$