Section 7.4: Integration by Partial Fractions

Suppose $f(x) = \dfrac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials *and* the degree of $h(x)$ is **greater** than the degree of $g(x)$ (if this is not true, you must first do long division). In order to integrate a partial fraction problem, you must first find the Partial Fraction Decomposition:

First **FACTOR** the denominator BEFORE setting up the decomposition

• **Case I**: The denominator is a product of *linear* factors, *none* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-2)(2x-11)} = \frac{A}{x-2} + \frac{B}{2x-11}$$

• **Case II**: The denominator is a product of *linear* factors, *some* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-1)(x-2)^3} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

• **Case III**: The denominator contains *irreducible quadratic* factors, *none* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-2)^2(x^2+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+1}$$

• **Case IV**: The denominator contains *irreducible quadratic* factors, *some* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+5}{(x-2)(x^2+4)^2} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

Once you find A, B, etc., then you integrate the result.

A common integral: $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

1. $\displaystyle\int \frac{1}{2x^3 + 5x^2 + 3x}\, dx =$

2. $\displaystyle\int_{-1}^{0} \frac{x^3}{x+2}\, dx =$

3. $\displaystyle\int \frac{1}{(x-1)^2(x+4)}\,dx =$

4. $\displaystyle\int \frac{x^4 - 11x^2 + 14x + 8}{x^3 - 4x^2 + 4x}\,dx =$

5. $\displaystyle\int \frac{3x^3+18}{x^2(x^2+9)}\,dx =$

6. $\displaystyle\int_0^1 \frac{x+6}{(x^2+1)(x^2+4)}\,dx =$