Section 12.2: Vectors in Three Dimension

A three dimensional vector is an ordered triple $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$. The numbers a_1, a_2 and a_3 are called the **components** of the vector $\overrightarrow{a}$. If a vector starts at the point $A(a_1, a_2, a_3)$ and ends at the point $B(b_1, b_2, b_3)$, then the vector with representation $\overrightarrow{AB}$ is given by $\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$.

Example 1: Find the components of the vector with initial point $A(-2, 4, 1)$ and terminal point $B(2, 2, -1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

The Algebra of Vectors: Suppose $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ and $\overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$ are vectors and c is a scalar.

a.) Scalar Multiplication: $c\overrightarrow{a} = \langle ca_1, ca_2, ca_3 \rangle$. (Changes magnitude if $c \neq \pm 1$ and direction if $c < 0$).

b.) Vector Sum: $\overrightarrow{a+b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$. ('Tip to tail')

c.) Vector Difference: $\overrightarrow{a-b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$. ('Tail to tail')

d.) Vector Length: $|\overrightarrow{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$.

e.) Unit Vector: A unit vector in the direction of $\overrightarrow{a}$ is $\overrightarrow{u} = \dfrac{\overrightarrow{a}}{|\overrightarrow{a}|}$. We call $\overrightarrow{i} = \langle 1, 0, 0 \rangle$, $\overrightarrow{j} = \langle 0, 1, 0 \rangle$ and $\overrightarrow{k} = \langle 0, 0, 1 \rangle$ **standard basis vectors**, and $\langle a_1, a_2, a_3 \rangle = a_1 \overrightarrow{i} + a_2 \overrightarrow{j} + a_3 \overrightarrow{k}$.

Example 2: Given $\overrightarrow{a} = \langle 1, 2, -1\rangle$ and $\overrightarrow{b} = \langle 0, 3, -5\rangle$, find:

a.) $\overrightarrow{a + 2b}$

b.) $|\overrightarrow{a - b}|$

c.) A unit vector in the direction of $\overrightarrow{b}$.

d.) A vector in the direction of $\overrightarrow{a + b}$ with length 7.

Example 3: Show the vectors $2\overrightarrow{i} + 6\overrightarrow{j} - 4\overrightarrow{k}$ and $-3\overrightarrow{i} - 9\overrightarrow{j} + 6\overrightarrow{k}$ are parallel.