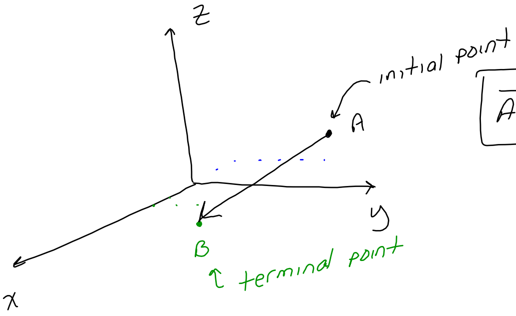
Section 12.2: Vectors in Three Dimension

A three dimensional vector is an ordered triple $\vec{a} = \langle a_1, a_2, a_3 \rangle$. The numbers a_1, a_2 and a_3 are called the **components** of the vector $\vec{a}$. If a vector starts at the point $A(a_1, a_2, a_3)$ and ends at the point $B(b_1, b_2, b_3)$, then the vector with representation $\overrightarrow{AB}$ is given by $\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$.

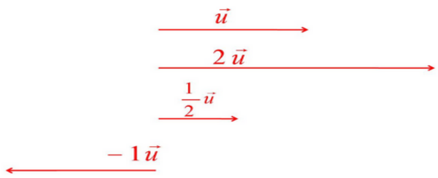
Example 1: Find the components of the vector with initial point $A(-2, 4, 1)$ and terminal point $B(2, 2, -1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.



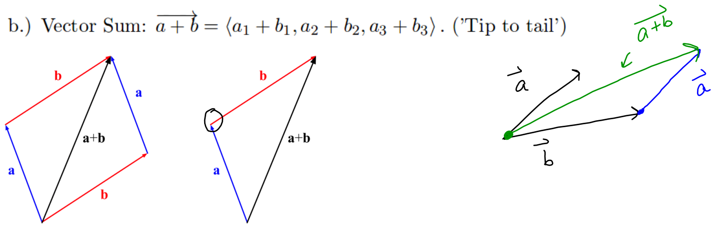
$\boxed{\overrightarrow{AB} = \langle 4, -2, -2 \rangle}$

The Algebra of Vectors: Suppose $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ are vectors and c is a scalar.

a.) Scalar Multiplication: $c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$. (Changes magnitude if $c \neq \pm 1$ and direction if $c < 0$).

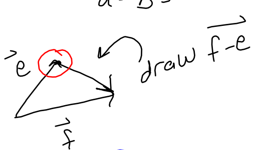


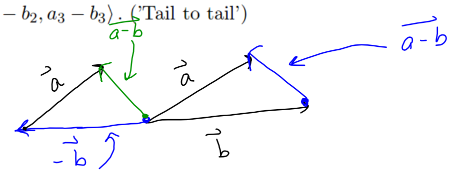
b.) Vector Sum: $\overrightarrow{a+b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$. ('Tip to tail')

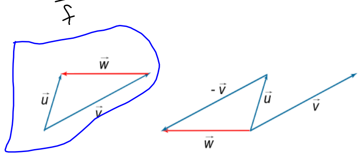


c.) Vector Difference: $\overrightarrow{a-b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$. ('Tail to tail')

$\overrightarrow{a-b} = \vec{a} + -\vec{b}$

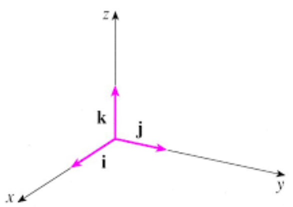






d.) Vector Length: $|\vec{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$.

e.) Unit Vector: A unit vector in the direction of $\vec{a}$ is $\vec{u} = \dfrac{\vec{a}}{|\vec{a}|}$. We call $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$ and $\vec{k} = \langle 0, 0, 1 \rangle$ **standard basis vectors**, and $\cancel{\langle a_1, a_2, a_3 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}}$.



Example 2: Given $\vec{a} = \langle 1, 2, -1\rangle$ and $\vec{b} = \langle 0, 3, -5\rangle$, find:

a.) $\overrightarrow{a + 2b}$

$$\vec{a} + 2\vec{b} = \langle 1, 2, -1\rangle + 2\langle 0, 3, -5\rangle$$

$$= \langle 1, 2, -1\rangle + \langle 0, 6, -10\rangle$$

$$\boxed{\overrightarrow{a+2b} = \langle 1, 8, -11\rangle}$$

b.) $|\overrightarrow{a - b}| = |\langle 1, 2, -1\rangle - \langle 0, 3, -5\rangle|$

$$= |\langle 1, -1, 4\rangle|$$

$$= \sqrt{1 + 1 + 16}$$

$$= \boxed{\sqrt{18}}$$

c.) A unit vector in the direction of $\vec{b}$.

$$\vec{u} = \frac{\vec{b}}{|\vec{b}|} = \frac{\langle 0, 3, -5\rangle}{\sqrt{9+25}} = \left\langle 0, \frac{3}{\sqrt{34}}, \frac{-5}{\sqrt{34}}\right\rangle$$

d.) A vector in the direction of $\overrightarrow{a + b}$ with length 7.

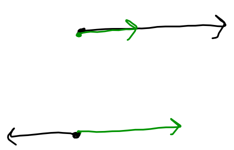
$a = \langle 1, 2, -1\rangle$, $\vec{b} = \langle 0, 3, -5\rangle$

$$\vec{u} = \frac{\overrightarrow{a+b}}{|a+b|} = \frac{\langle 1, 5, -6\rangle}{\sqrt{1+25+36}}$$

$$\vec{u} = \frac{\langle 1, 5, -6\rangle}{\sqrt{62}}$$

$$\boxed{7\vec{u} = \frac{7}{\sqrt{62}}\langle 1, 5, -6\rangle}$$

Example 3: Show the vectors $2\vec{i} + 6\vec{j} - 4\vec{k}$ and $-3\vec{i} - 9\vec{j} + 6\vec{k}$ are parallel.



Two vectors are parallel if they are scalar multiples of each other.

$$c\langle 2, 6, -4\rangle = \langle -3, -9, 6\rangle$$

$$2c = -3$$

$$c = -\frac{3}{2}$$

$$-\frac{3}{2}\langle 2, 6, -4\rangle = \langle -3, -9, 6\rangle \checkmark$$