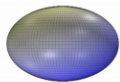


Section 12.6: Quadric Surfaces

**Definition:** A **quadric surface** is the graph of a **second degree** equation in three variables.

I. Ellipsoid: The quadric surface with the equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  is called an **ellipsoid**



II. Hyperboloid:

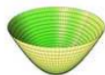
A.) Hyperboloid of one sheet:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ .



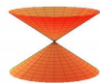
B.) Hyperboloid of two sheets:  $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .



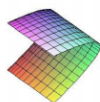
III. Elliptic Paraboloid:  $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ .



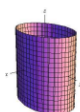
IV. Cone:  $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ .



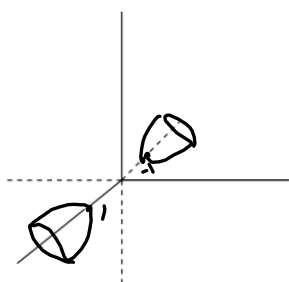
V. Parabolic cylinder: One general form is  $y = ax^2$ . Sketch the graph of  $y = ax^2$  in the  $xy$ - plane, then raise and lower along the  $z$  axis.



VI. Elliptic cylinder: One general form is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

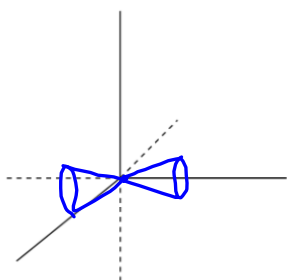


Example 1: Sketch the graph of  $x^2 - \frac{y^2}{4} - \frac{z^2}{9} = 1$  and label the intercepts.

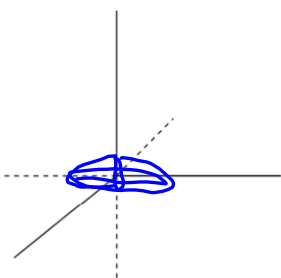


Two sheet hyperboloid  
 $x$ -int  $x = \pm 1$

Example 2: Sketch the graph of  $y^2 = \frac{x^2}{9} + \frac{z^2}{4}$  and label the intercepts.

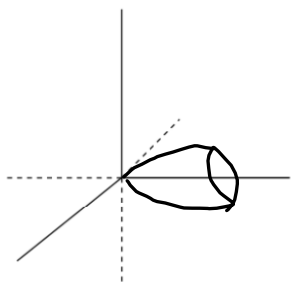


Example 3: Sketch the graph of  $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$  and label the intercepts.



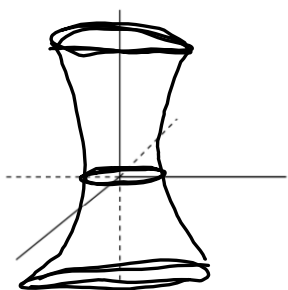
ellipsoid  
 $x$ -int  $\pm 1$   
 $y$ -int  $\pm 3$   
 $z$ -int  $\pm 2$

Example 4: Sketch the graph of  $y = \frac{x^2}{4} + \frac{z^2}{16}$  and label the intercepts.



elliptic paraboloid

Example 5: Sketch the graph of  $\frac{y^2}{4} + \frac{x^2}{9} = 1 + z^2$  and label the intercepts.



$$\frac{y^2}{4} + \frac{x^2}{9} - z^2 = 1$$

Hyperboloid one sheet

Example 6: Sketch the graph of  $x^2 + \frac{z^2}{9} - 4 = 0$

