Section 13.4: Motion in Space: Velocity and Acceleration

Definition: Suppose a particle moves through space so that its position at time t is given by $\mathbf{r}(t)$.

(i) The velocity of the particle at time t is $\mathbf{v}(t) = \mathbf{r}'(t)$.

(ii) The speed of the particle at time t is $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$.

(iii) The acceleration of the particle at time t is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

Example 1: Find the velocity, acceleration, and speed of a particle with position function $\mathbf{r}(t) = \langle e^t, e^{-t} \rangle$ at $t = 0$. Find velocity and acceleration vectors for the given value of t.

$v(t) = r'(t) = \langle e^t, -e^{-t} \rangle$ $a(t) = \langle e^t, e^{-t} \rangle$

$\boxed{v(0) = \langle 1, -1 \rangle}$ $\boxed{a(0) = \langle 1, 1 \rangle}$

Example 2: Find the velocity, acceleration and speed of a particle with position function $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, t \rangle$. at $t=0$

$v(t) = \langle e^t \cos t - e^t \sin t, e^t \sin t + e^t \cos t, 1 \rangle$

you do $a(t)$

$v(0) = \langle 1, 1, 1 \rangle$

speed is $|v(0)| = \sqrt{3}$

Example 3: The acceleration of a particle at time t is given by $\mathbf{a}(t) = \langle t, t^2, \cos(2t) \rangle$. Given that $\mathbf{v}(0) = \langle 1, 0, 1 \rangle$ and $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$, find $\mathbf{r}(t)$.

$t=0$ $v(t) = \langle \frac{t^2}{2} + c_1, \frac{t^3}{3} + c_2, \frac{1}{2}\sin(2t) + c_3 \rangle$

$\langle c_1, c_2, c_3 \rangle = \langle 1, 0, 1 \rangle$

$c_1 = 1$
$c_2 = 0$
$c_3 = 1$

$c_1 = 1$
$c_2 = 0$
$c_3 = 1$

$v(t) = \langle \frac{t^2}{2} + 1, \frac{t^3}{3}, \frac{1}{2}\sin(2t) + 1 \rangle$

$r(t) = \langle \frac{t^3}{6} + t + \cancel{c_1}^{0}, \frac{t^4}{12} + \cancel{c_2}^{1}, -\frac{1}{4}\cos(2t) + t + \cancel{c_3}^{\frac{1}{4}} \rangle$

$c_1 = 0$, $c_2 = 1$, $-\frac{1}{4} + c_3 = 0$

$c_3 = \frac{1}{4}$