Section 14.1 Functions of Several Variables

Definition: Let D be a subset of R^2. A **function** f **of two variables** is a rule that assigns to each ordered pair (x, y) in D a unique real number $f(x, y)$. The set D is called the **domain** of f and its **range** is the set of values $\{f(x, y) | (x, y) \in D\}$. Note: Just as the domain of $y = f(x)$ is the set of all values of x for which $f(x)$ is defined, the domain of $z = f(x, y)$ is the set of all **points** (x, y) for which $f(x, y)$ is defined. **Thus the domain of** $z = f(x, y)$ **lies entirely in the** xy**-plane.**

Example 1: Find and sketch the domain of the following functions.

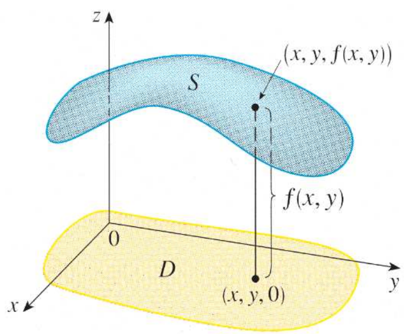
a.) $f(x, y) = \ln(2x + y)$

b.) $f(x, y) = \sqrt{x} + \sqrt{y}$

c.) $f(x, y) = \dfrac{\sqrt{9 - x^2 - y^2}}{x + 2y}$

Definition: If f is a function of two variables with domain D, the **graph** of f is the set

$S = \{(x, y, z) \in R^3 | z = f(x, y), (x, y) \in D\}$. Just as the graph of a function of one variable is a curve with equation $y = f(x)$, the graph of a function f of two variables is a **surface** S with equation $z = f(x, y)$. We can visualize the graph S of f as lying directly above or below its domain D in the xy plane.



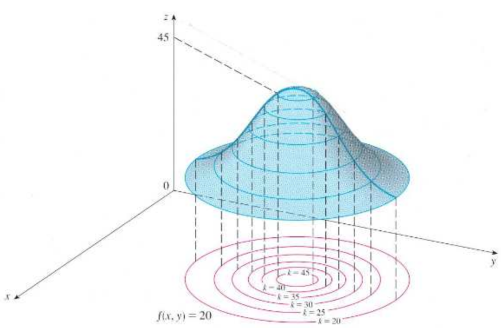
Example 2: Sketch the graph of:

a.) $f(x, y) = x + 2y + 4$

b.) $f(x, y) = x^2 + 9y^2$

c.) $f(x, y) = \sqrt{x^2 + y^2}$

Definition: The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f. In other words, a level curve shows where the graph of f has height k. The level curves $f(x, y)$ are just the horizontal traces of the graph of f in the plane $z = k$ projected down to the xy plane. A graph of the level curves is called a **countour plot**.

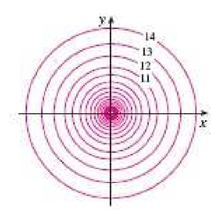


Example 3: Describe the level curves of

a.) a hyperboloid of one sheet

b.) a hyperboloid of two sheets

Example 4: For the level curve shown below, sketch the surface.



Example 5: Sketch the level curves for the following functions:

a.) $f(x, y) = 2 + 4x - y$ for $k = -2, 0, 2$.

b.) $f(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$.

Definition: A **function of three variables**, f, is a rule that assigns to each ordered triple (x, y, z) in the doman $D \subset R^3$ a real number $w = f(x, y, z)$. The graph of a function of three variables is in four dimensional space. The domain of a function of three variables is in three dimension.

Example 6: Find the domain of $f(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2 - 9}}$.

A function of three variables has **level surfaces**, which are surfaces of the form $f(x, y, z) = k$. If the point $f(x, y, z)$ moves along a level surface, the value $f(x, y, z)$ remains fixed.

Example 7: Describe the level surfaces of $f(x, y, z) = x + y + z$.

Example 8: Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.