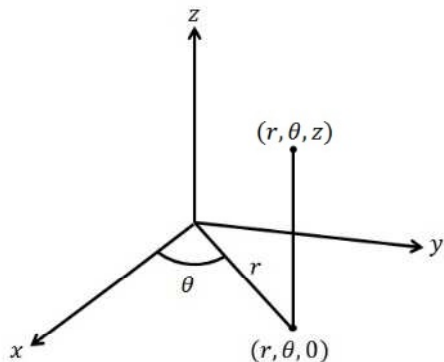


Section 15.7 Integration in Cylindrical Coordinates

Recall that in section 15.3 we introduced integration in the polar coordinate system. In three dimensions there are two coordinate systems that are similar to polar coordinates and give convenient descriptions of some commonly occurring surfaces and solids.

In the **Cylindrical Coordinate System**, a point P is represented by an ordered triple (r, θ, z) where r and θ are polar coordinates of the projection of the point P on onto the xy -plane and z is the directed distance from the xy -plane to P .



To convert from cylindrical to rectangular coordinates, we use the equations $x = r \cos \theta$, $y = r \sin \theta$, $z = z$. To convert from rectangular to cylindrical coordinates, use the equations $x^2 + y^2 = r^2$, $\tan \theta = \frac{y}{x}$, $z = z$.

Example 1: Find the cylindrical coordinates of the rectangular coordinate $(\sqrt{3}, 1, 4)$.

Example 2: Find the rectangular coordinates of the cylindrical coordinate $\left(2, \frac{2\pi}{3}, 5\right)$

Example 3: Write the equation in cylindrical coordinates

a.) $x^2 + y^2 + z^2 = 25$

b.) $z = 12 - 4x^2 - 4y^2$

c.) $z = \sqrt{3x^2 + 3y^2}$

Triple Integrals in Cylindrical Coordinates

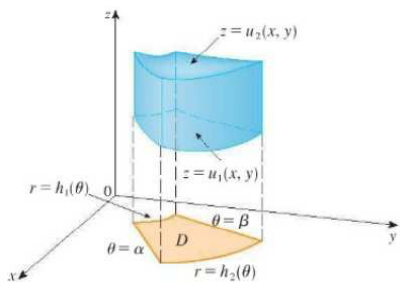
Suppose that E is a solid whose image D on the xy -plane can be described in **polar coordinates**:

$E = \{(x, y, z) | (x, y) \in D, \phi_1(x, y) \leq z \leq \phi_2(x, y)\}$ where D is given in polar coordinates by

$D = (r, \theta) | \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\}$.

We know from section 15.6 that $\iiint_E f(x, y, z) dV = \iint_D \left[\int_{\phi_1(x, y)}^{\phi_2(x, y)} f(x, y, z) dz \right] dA$. Combining this with what we know from section 15.3, we get

$$\iiint_E f(x, y, z) dV = \int_{\theta=\alpha}^{\theta=\beta} \int_{r=h_1(\theta)}^{r=h_2(\theta)} \int_{z=\phi_1(r \cos \theta, r \sin \theta)}^{z=\phi_2(r \cos \theta, r \sin \theta)} f(r \cos \theta, r \sin \theta, z) dz r dr d\theta$$



Example 4: Evaluate $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^3 (x^2 + y^2) dz dy dx$ by converting to cylindrical coordinates.

Example 5: Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{4-x^2-y^2} yz dz dx dy$ by converting to cylindrical coordinates.

Example 6: Evaluate $\iiint_E y \, dV$ where E is the bounded by $z = 4 - x^2 - y^2$ in the first octant.

Example 7: Find the volume of the solid E bounded by $x^2 + y^2 + z^2 = 9$ and $8z = x^2 + y^2$.

Example 8: Find $\iiint_E \sqrt{x^2 + y^2} dV$ if E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$.

Example 9: Evaluate $\iiint_E xy dV$ where E is the solid that lies with the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ below the cone $z = \sqrt{9x^2 + 9y^2}$.