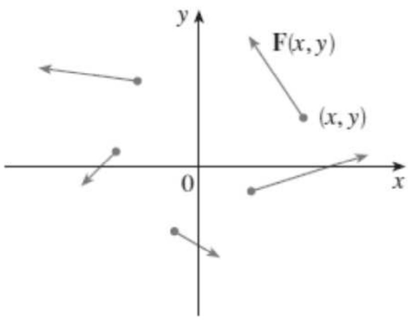
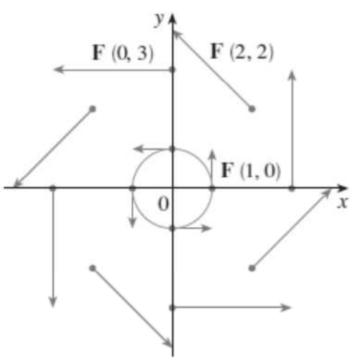
Section 16.1 Vector Fields

Definition: A **vector field** in two dimension is a function **F** that assigns to each point (x, y) in $D \subset \mathbb{R}^2$ a two dimensional vector, $\mathbf{F}(x, y)$.

In two dimension, the vector field lies entirely in the xy plane.

A few vector fields in $\mathbb{R}^2$:

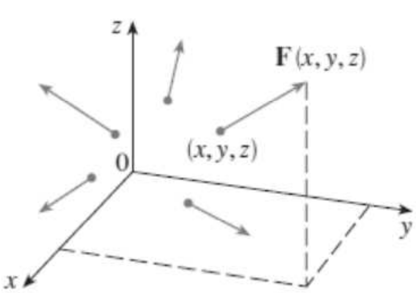




Definition: A **vector field** in three dimension is a function **F** that assigns to each point (x, y, z) in $D \subset \mathbb{R}^3$ a three dimensional vector, $\mathbf{F}(x, y, z)$.

In three dimension, the vector field is in space.

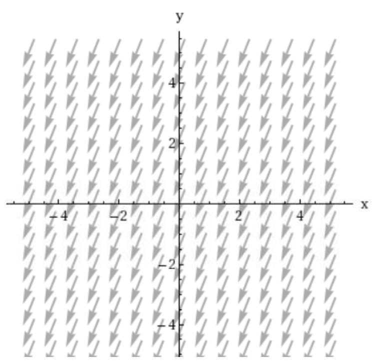
A vector field in $\mathbb{R}^3$:



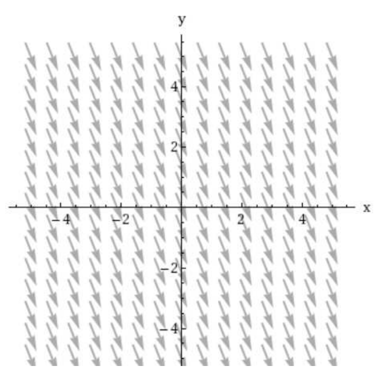
In order to match $\mathbf{F}$ with it's vector field, choose a several points, (x, y), in each quadrant, and look at the *direction* of $\mathbf{F}(x, y)$. Often times, it is a process of elimination.

Example 1: Which of the following is the vector field for $\mathbf{F}(x, y) = \langle 2x, -7 \rangle$?

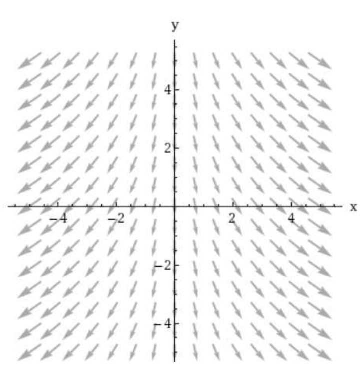
a.)



b.)



c.)



Recall from chapter 14: The **gradient** of a function $f(x, y)$ is $\nabla f = \langle f_x(x, y), f_y(x, y)\rangle$. **Thus we can now think of the gradient as being a vector field**.

Example 2: Find the gradient of $f(x, y) = \sqrt{x^2 + y^2}$.

Example 3: Find the gradient of $f(x, y, z) = x \ln(y - z)$.