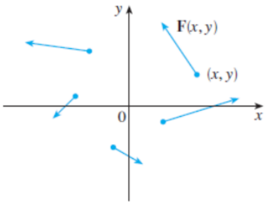
## Section 16.1 Vector Fields

Definition: A **vector field** in two dimension is a function $\mathbf{F}$ that assigns to each point $(x, y)$ in $D \subset \mathbb{R}^2$ a two dimensional vector, $\mathbf{F}(x, y)$.

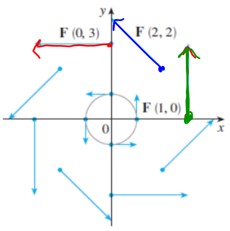
In two dimension, the vector field lies entirely in the $xy$ plane.

A few vector fields in $\mathbb{R}^2$:




$\mathbf{F}(x, y) = -y\,\mathbf{i} + x\,\mathbf{j}$

$F(2,2) = \langle -2, 2 \rangle$



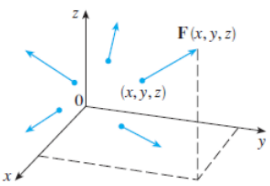

$F(0,3) = \langle -3, 0 \rangle$

$F(2,0) = \langle 0, 2 \rangle$

Definition: A **vector field** in three dimension is a function $\mathbf{F}$ that assigns to each point $(x, y, z)$ in $D \subset \mathbb{R}^3$ a three dimensional vector, $\mathbf{F}(x, y, z)$.

In three dimension, the vector field is in space.

A vector field in $\mathbb{R}^3$:

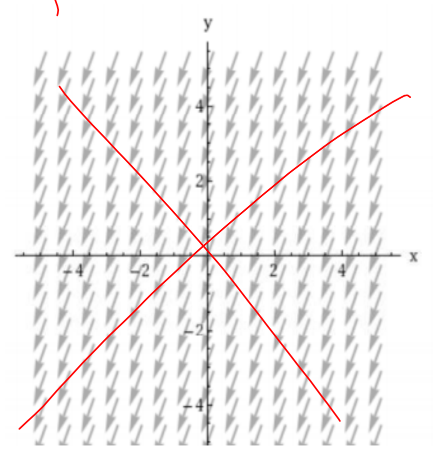


In order to match **F** with it's vector field, choose a several points, $(x, y)$, in each quadrant, and look at the *direction* of $\mathbf{F}(x, y)$. Often times, it is a process of elimination.

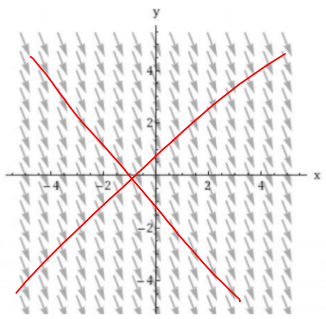
Example 1: Which of the following is the vector field for $\mathbf{F}(x, y) = \langle 2x, -7 \rangle$?

$F(1,1) = \langle 2, -7 \rangle$

~~a.)~~




b.)

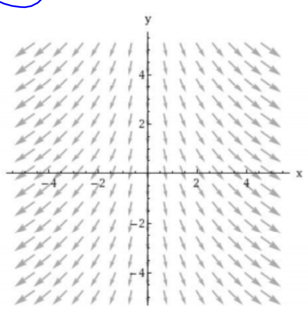



$F(x,y) = \langle 2x, -7 \rangle$

$F(1,1) = \langle 2, -7 \rangle$

$F(-1,1) = \langle -2, -7 \rangle$

c.)




$F(x,y) = \langle 2x, -7 \rangle$

$F(-1,1) = \langle -2, -7 \rangle$

$F(-1,-1) = \langle -2, -7 \rangle$

$F(1,-1) = \langle 2, -7 \rangle$

Recall from chapter 14: The **gradient** of a function $f(x,y)$ is $\boxed{\nabla f = \langle f_x(x,y), f_y(x,y)\rangle}$. **Thus we can now think of the gradient as being a vector field.**

Example 2: Find the gradient of $f(x,y) = \sqrt{x^2+y^2}$.

① we call $f(x,y)$ a scalar function

② we call $\vec{F}(x,y)$ a vector function

$f(x,y) = \sqrt{x^2+y^2}$

$\nabla f = \langle f_x, f_y \rangle$

$f(x,y) = (x^2+y^2)^{\frac{1}{2}}$

$f_x = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}}(\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}}(\cancel{2}y) = \frac{y}{\sqrt{x^2+y^2}}$

$\nabla f = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$

Example 3: Find the gradient of $f(x,y,z) = x\ln(y-z)$.

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = (1)\ln(y-z)$

$f_y = (x)\left(\frac{1}{y-z}\right)$

$f_z = (x)\left(\frac{-1}{y-z}\right)$

$$\boxed{\nabla f = \left\langle \ln(y-z), \frac{x}{y-z}, \frac{-x}{y-z} \right\rangle}$$