

Section 11.2 (more on geometric series)

9. Consider $\sum_{n=1}^{\infty} (x-5)^n$. Find the value(s) of x for which the series converges. Find the sum of the series for those values of x .

Recall: Geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad \boxed{\text{if } |r| < 1}$$

$$= \frac{\text{"first term"}}{1-r}$$

$$\text{For } \sum_{n=1}^{\infty} \frac{(x-5)^n}{r^n}, \quad |x-5| < 1$$

$$\begin{aligned} \text{will converge} \\ \text{on } (4, 6) \end{aligned}$$

$$\sum_{n=1}^{\infty} (x-5)^n = \sum_{n=1}^{\infty} \frac{(x-5)(x-5)^{n-1}}{a} = \frac{a}{1-r} \quad \text{sum} \rightarrow$$

$$\frac{\text{"First term"}}{1-r} = \frac{x-5}{1-(x-5)} = \frac{x-5}{-x+6} \quad \text{SAME!}$$

$$\sum_{n=2}^{\infty} (-3) \left(\frac{2}{7}\right)^{n-1} \quad \begin{aligned} \text{does the series converge?} \\ \text{what is the sum?} \end{aligned}$$

$$r = -\frac{6}{7} \quad \sum_{n=2}^{\infty} (-3)(-3) \left(\frac{2}{7}\right)^{n-1} \quad a^nb^n = (ab)^n$$

$$\sum_{n=2}^{\infty} (-3) \left(-\frac{6}{7}\right)^{n-2}$$

$$\sum_{n=2}^{\infty} (-3) \left(-\frac{6}{7}\right) \left(-\frac{6}{7}\right)$$

$$\sum_{n=2}^{\infty} \frac{18}{7} \left(-\frac{6}{7}\right)^{n-2} = \frac{a}{1-r} = \frac{\frac{18}{7}}{1 + \frac{6}{7}}$$

$$\sum_{n=2}^{\infty} (-3) \left(\frac{2}{7}\right)^{n-1}, \quad \frac{\text{First term}}{1-r} = \frac{(9) \left(\frac{2}{7}\right)}{1 + \frac{6}{7}} \quad \text{SAME!!}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-2} \left(\frac{1}{2}\right) \left(2\right)$$

$$\sum_{n=1}^{\infty} 2 \left(\frac{1}{2}\right)^{n-1} = \frac{2}{1 - \frac{1}{2}}$$

more on remainder estimate for integral test
 Given $\sum_{n=1}^{\infty} a_n$ converges, and S_n is used to approximate the sum.
 $S_n = a_1 + a_2 + \dots + a_n$

The remainder $R_n = \text{sum} - S_n$
 aka "error"
 A bound on the remainder (error)
 is $R_n < \int_n^{\infty} f(x) dx$

$$\text{For } \sum_{n=1}^{\infty} \frac{1}{n^n}$$

(a) use S_6 to approximate the sum

S_6 = sum of first six terms

$$S_6 = 1 + \frac{1}{2^6} + \frac{1}{3^6} + \frac{1}{4^6} + \frac{1}{5^6} + \frac{1}{6^6}$$

(b) Use the Remainder Estimate for the Integral Test to estimate the remainder (error) in using the ~~partial sum~~ partial sum to approximate the sum of the series. (Round your answer to six decimal places if necessary.)

$$R_n < \int_n^{\infty} \frac{dx}{x^n} \quad \text{here } n=6.$$

$$\begin{aligned} R_6 &< \int_6^{\infty} \frac{dx}{x^6} = \lim_{t \rightarrow \infty} \int_6^t \frac{dx}{x^6} \\ &= \lim_{t \rightarrow \infty} \left[-\frac{1}{5x^5} \right]_6^t \\ &= \lim_{t \rightarrow \infty} \left(-\frac{1}{5t^5} + \frac{1}{5 \cdot 6^5} \right) \\ &= \frac{1}{10(6^5)} \end{aligned}$$

(c) Using the Remainder Estimate for the Integral Test, find a value of n that will ensure that the error in the approximation s_n is less than 0.00001.

$$\begin{aligned} \text{we must find } n \text{ so that } & \text{error (remainder)} \\ a < b & \rightarrow a < c \\ R_n &< \int_n^{\infty} \frac{dx}{x^n} < .00001 \quad \text{is less than .00001.} \\ &\rightarrow R_n < .00001 \end{aligned}$$

$$\begin{aligned} \lim_{t \rightarrow \infty} \int_n^t \frac{dx}{x^n} &= \lim_{t \rightarrow \infty} \left[-\frac{1}{(10x)^{10}} \right]_n^t \\ &= \lim_{t \rightarrow \infty} \left(-\frac{1}{10t^{10}} + \frac{1}{10n^{10}} \right) \\ &= \frac{1}{10n^{10}} < .00001 \\ \frac{1}{10(.00001)} &< n \\ \sqrt[10]{.00001} &< n \\ \downarrow & \\ 2.5 &< n \quad \text{since } n \text{ must be an integer} \\ n > 3 & \end{aligned}$$

use the remainder estimate for the integral test to find the smallest value of n so that S_n approximates

$$\sum_{n=1}^{\infty} \frac{3}{n^3} \text{ to within } \frac{1}{100}.$$

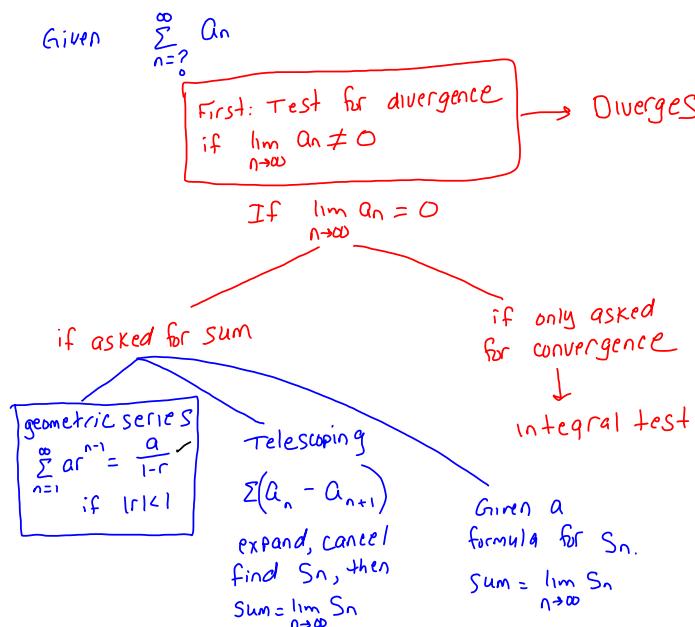
$$\begin{aligned} R_n &< \int_n^{\infty} \frac{3}{x^3} dx \\ &\rightarrow \lim_{t \rightarrow \infty} \int_n^t \frac{3}{x^3} dx = \lim_{t \rightarrow \infty} \left[-\frac{3}{2x^2} \right]_n^t \\ &= \lim_{t \rightarrow \infty} \left(-\frac{3}{2t^2} + \frac{3}{2n^2} \right) \end{aligned}$$

$$\frac{3}{2n^2} < \frac{1}{100} \quad = \frac{3}{2n^2}$$

$$\frac{300}{2} < n^2$$

$$150 < n^2 \quad n=13$$

Flow chart for sections 11.2 & 11.3



Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{e^{1/n^5}}{n^6}$$

$$\text{TD } \lim_{n\rightarrow\infty} \frac{e^{1/n^5}}{n^6} = \frac{0}{\infty} = 0$$

test fails!

$$\int_1^{\infty} \frac{e^{1/x^5}}{x^6} dx = \lim_{t\rightarrow\infty} \int_1^t \frac{e^{1/x^5}}{x^6} dx$$

$$u = \frac{1}{x^5} \quad \begin{cases} x=t, u=\frac{1}{t^5} \\ x=1, u=1 \end{cases}$$

$$du = -5x^{-6} dx$$

$$du = \frac{-5}{x^6} dx$$

$$\lim_{t\rightarrow\infty} \int_1^t -\frac{1}{5} e^{1/u} du = \lim_{t\rightarrow\infty} -\frac{1}{5} e^{1/u} \Big|_1^t$$

$$\lim_{t\rightarrow\infty} -\frac{1}{5} \left(e^{1/t^5} - e^1 \right) = -\frac{1}{5} (1-e)$$

integral converges,
so does series

$$\sum_{n=1}^{\infty} \left(e^{1/n^5} - e^{1/(n+2)^5} \right)$$

Find sum. n and n+2
are not consecutive

$$\textcircled{1} \quad S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n + a_{n+1} + a_{n+2}$$

$$S_n = e^{1/1^5} - e^{1/2^5} + e^{1/2^5} - e^{1/3^5} + e^{1/3^5} - e^{1/4^5} + \dots + e^{1/(n-1)^5} - e^{1/n^5} + e^{1/n^5} - e^{1/(n+2)^5}$$

$$S_n = e^{1/1^5} - e^{1/(n+2)^5}$$

$$\text{Sum: } \lim_{n\rightarrow\infty} S_n = \lim_{n\rightarrow\infty} \left(e^{1/1^5} - e^{1/(n+2)^5} \right)$$

$$= \boxed{e^{1/1^5} - e^{1/(n+2)^5}}$$

