**Spring 2015 Math 151**  
Sample Questions for Exam 3  
courtesy: Amy Austin

Review Exercises: Sections 4.3 - 6.1

**Section 4.3**

1. Evaluate \( \log_3 108 - \log_3 4 \)
2. Solve for \( x \): \( \log(x + 3) + \log(x) = 1 \)
3. Solve for \( x \): \( \ln x - \ln(x + 1) = \ln 2 + \ln 3 \)
4. Find \( \lim_{x \to 2^+} \ln(x - 2) \)
5. Find \( \lim_{x \to \infty} [\log(2x^2 - 1) - \log(3x^2 + 6)] \)
6. What is the domain of \( f(x) = \ln(x^2 + 2x - 8) \)?

**Section 4.4**

7. Find \( f'(x) \) for \( f(x) = \ln(2x^2 - 8) \)
8. Find the derivative of \( f(x) = 2\cos x + \log_7(3x - 1) \)
9. Find \( y' \) for \( y = (\cos x)^{\tan x} \)
10. Find the slope of the tangent line to the curve \( f(x) = x \ln(x) \) at \( x = e^2 \).

**Section 4.5**

11. At a certain instant, 100 grams of a radioactive substance is present. After 4 years, 20 grams remain.
   a.) What is the half life of the substance?
   b.) How much of the substance remains after 2.5 years?
12. A bowl of soup at temperature 180\(^\circ\) is placed in a 70\(^\circ\) room. If the temperature of the soup is 150\(^\circ\) after 2 minutes, when will the soup be an eatable 100\(^\circ\)?

**Section 4.6**

13. Express \( \tan(\arcsin x) \) as an algebraic expression.
14. Find the derivative of \( y = x^2 \arccos(e^{3x}) \)
15. Find the equation of the line tangent to \( y = \tan^{-1}(2x - 1) \) when \( x = 1 \).
16. Compute the exact value of \( \lim_{x \to \infty} \arccos \left( \frac{1 + 2x}{3 - 4x} \right) \)
17. Compute \( \sec(\arctan(-\sqrt{5})) \)
18. Compute \( \arcsin(\sin 4\pi/3) \)

**Section 4.8**

19. Find the limits of each of the following:
   a.) \( \lim_{x \to 0} \frac{\arcsin(3x)}{2x} \)
   b.) \( \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} \)
   c.) \( \lim_{x \to 0^+} \frac{\ln x}{\sqrt{x}} \)
   d.) \( \lim_{x \to \pi/2^-} (\sec x - \tan x) \)
   e.) \( \lim_{x \to 1^+} (x - 1) \tan(\pi x/2) \)
   f.) \( \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^{4x} \)
Section 5.1 - 5.3

20. If \( f(x) = \frac{1}{x} \), verify \( f(x) \) satisfies the Mean Value Theorem on the interval \([1, 10]\) and find all \( c \) that satisfies the conclusion of the Mean Value Theorem.

21. Find the absolute maximum and minimum of the given function on the given interval.
   a) \( x^3 - 5x^2 + 3 \) on \([-1, 3]\)
   b) \( x \ln x \) on \([1, e]\)

22. Find the intervals where the given function is increasing and decreasing, local extrema, intervals of concavity and inflection points.
   a) \( f(x) = x^3 - 2x^2 + x \)
   b) \( f(x) = xe^{2x} \)

23. Find the concavity of \( f \) if \( f'(x) = \frac{\ln x}{x} \)

24. In the graph that follows, the graph of \( f' \) is given. Using the graph of \( f' \), determine all critical values of \( f \), where \( f \) is increasing and decreasing, local extrema of \( f \), where \( f \) is concave up and concave down, and the \( x \)-coordinates of the inflection points of \( f \). Assume \( f \) is continuous.

26. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length 3 m and 4 m if two sides of the rectangle lie along the legs.

Section 5.7

27. Find an antiderivative of \( \frac{1}{\sqrt{1 - x^2}} - \frac{1 + x}{x} \).

28. Given \( f''(x) = 2e^x - 4\sin(x) \), \( f(0) = 1 \), and \( f'(0) = 2 \), find \( f(x) \).

29. Find the vector functions that describe the velocity and position of a particle that has an acceleration of \( a(t) = \langle \sin t, 2 \rangle \), initial velocity of \( v(0) = \langle 1, -1 \rangle \) and an initial position of \( r(0) = \langle 0, 0 \rangle \).

Section 6.1

30. \( \sum_{i=2}^{5} i^2 = \)

31. Write \( 1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5} \) in summation notation.

32. \( \sum_{i=3}^{99} \left( \frac{1}{i} - \frac{1}{i+1} \right) = \)

Section 5.5

25. A cardboard rectangular box holding 32 cubic inches with a square base and open top is to be constructed. If the material for the base costs $2 per square inch and material for the sides costs $5 per square inch, find the dimensions of the cheapest such box.