Section 2.3

Compute the exact value of the following limits. If the limit does not exist, support your answer by evaluating left and right hand limits.

1. \[ \lim_{{x \to 1}} (4x^3 - 3x + 1) \]
2. \[ \lim_{{x \to -5}} \frac{x^2 + 5x}{x + 5} \]
3. \[ \lim_{{x \to 2}} \frac{x - \sqrt{3x - 2}}{x^2 - 4} \]
4. \[ \lim_{{h \to 0}} \frac{(3 + h)^{-1} - 3^{-1}}{h} \]
5. \[ \lim_{{x \to 1}} \frac{x - 4}{x - 1} \]
6. \[ \text{lim}_{{x \to 3}} f(x), \text{ where } f(x) = \begin{cases} x + 5 & \text{if } x \leq 3 \\ x^3 - 3 & \text{if } x > 3 \end{cases} \]
7. \[ \lim_{{x \to 0}} \left( \frac{1}{x} - \frac{1}{|x|} \right) \]
8. \[ \lim_{{x \to 2}} \frac{x^2 - 4}{|x - 2|} \]
9. \[ \text{lim}_{{x \to 1}} f(x) \text{ if it is known that } 4x \leq f(x) \leq x + 3 \text{ for all } x \text{ in } [0, 2]. \]

Section 2.5

10. Referring to the graph, explain why the function \( f(x) \) is or is not continuous (you decide which) at \( x = -1, x = 3, x = 5, x = -4 \) and \( x = 7 \). For the values of \( x \) where \( f(x) \) is not continuous, is it continuous from the right, left or neither? In addition, for each discontinuity, is it a jump discontinuity, infinite discontinuity or a removable discontinuity?

11. Sketch the graph of \( f(x) \) and determine where the function \[ f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ 4x - 1 & -1 \leq x < 1 \\ 3 & x = 1 \\ 5 - x & \text{if } x > 1 \end{cases} \]
is not continuous. Fully support your answer.

12. Which of the following functions has removable discontinuity at \( x = a \)? If the discontinuity is removable, find a function \( g \) that agrees with \( f \) for \( x \neq a \) and is continuous at \( x = a \). Note: \( f \) has removable discontinuity at \( x = a \) if \( \lim_{{x \to a}} f(x) \) exists and \( f(x) \) can be redefined so that \( \lim_{{x \to a}} f(x) = f(a) \) (thereby removing the discontinuity).

(a) \( f(x) = \frac{x^2 - 4}{x - 2}, \quad x = 2 \).
(b) \( f(x) = \frac{1}{x - 1}, \quad x = 1 \).
(c) \( f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x + 4 & \text{if } x \geq 1 \end{cases}, \quad x = 1 \).

13. If \( f(x) = \frac{x + 2}{x^2 + 5x + 6} \), find all values of \( x = a \) where the function is discontinuous. For each discontinuity, find the limit as \( x \) approaches \( a \), if the limit exists. If the limit does not exist, support your answer by evaluating left and right hand limits.

14. Suppose it is known that \( f(x) \) is a continuous function defined on the interval \([1, 5]\). Suppose further it is given that \( f(1) = -3 \) and \( f(5) = 6 \). Give a graphical argument that there is at least one solution to the equation \( f(x) = 1 \).

15. If \( g(x) = x^5 - 2x^3 + x^2 + 2 \), use the Intermediate Value Theorem to find an interval which contains a root of \( g(x) \), that is contains a solution to the equation \( g(x) = 0 \).

16. Find the values of \( c \) and \( d \) that will make \[ f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases} \]
continuous on all real numbers. Once the value of \( c \) and \( d \) is found, find \( \lim_{{x \to 1}} f(x) \) and \( \lim_{{x \to 2}} f(x) \).
Section 2.6

17. Compute the following limits:

a.) \( \lim_{x \to \infty} \frac{4x^3 - 6x^4}{2x^3 - 9x + 1} \)

b.) \( \lim_{t \to -\infty} \frac{t^9 - 4t^{10}}{t^{12} + 2t^2 + 1} \)

c.) \( \lim_{x \to \infty} \frac{4x - 6x^3}{-2x^3 - 9x + 1} \)

d.) \( \lim_{x \to \infty} \frac{\sqrt{2 + x^2}}{4 - 7x} \)

e.) \( \lim_{x \to -\infty} \frac{\sqrt{5x^2 + 1}}{x - 3} \)

f.) \( \lim_{x \to \infty} \left( \frac{\sqrt{x^2 + 5x + 1}}{x} - x \right) \)

g.) \( \lim_{x \to -\infty} \left( x + \frac{\sqrt{x^2 + x + 2}}{x} \right) \)

18. Find all horizontal and vertical asymptotes of

\( f(x) = \frac{x^3}{x^3 - x} \)