Section 3.9

1. Given \( x = \cos t \) and \( y = t^2 \), find \( \frac{dy}{dx} \). Next, find the equation of the tangent line at \( t = \frac{\pi}{4} \).

2. Let \( x = t^4 - 4t^3 \) and \( y = 3t^2 - 6t \).

   a.) Find the equation of the tangent line at the point \((5, 9)\).

   b.) Find all point(s) on the curve where the tangent line is vertical or horizontal.

3. Show the curve \( x = \cos t \) and \( y = \sin t \cos t \) has two tangents at \((0, 0)\). Find the equations of these tangent lines.

4. At what points on the curve \( x = t^3 + 4t \), \( y = 6t^2 \) is the tangent line parallel to the line with equations \( x = -7t \), \( y = 12t - 5 \)?

Section 3.10

5. Water leaking onto a floor creates a circular pool with an area that increases at a rate of 3 square inches per minute. How fast is the radius of the pool increasing when the radius is 10 inches?

6. When a rocket is 2 miles high, it is moving vertically upward at a speed of 300 mph. At that instant, how fast is the angle of elevation of the rocket increasing, as seen by an observer on the ground 5 miles from the launching pad?

7. A filter in the shape of a cone is 6 inches high and has a radius of 2 inches at the top. A solution is poured in the cone so that the water level is rising at a rate of \( \frac{3}{2} \) inches per second. How fast is the water being poured in when the water level has a depth of 2 inches?

8. One end of a 13 foot ladder is on the ground, and the other end rests on a vertical wall. If the top of the ladder is being pushed up the wall at a rate of 1 foot per second, how fast is the base of the ladder approaching the wall when it is 3 feet from the wall?

9. A point moves around the circle \( x^2 + y^2 = 9 \). When the point is at \((-\sqrt{3}, \sqrt{6})\), its \( x \) coordinate is increasing at a rate of 20 units per second. How fast is its \( y \) coordinate changing at that instant?

10. Two sides of a triangle have lengths 5 m and 4 m. The angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is 60°.

11. A trough is 10 feet long and its ends have the shape of isosceles triangles that are 3 feet across the top and have a height of 1 foot. If the trough is filled with water at a rate of 12 cubic feet per minute, how fast is the water level rising when the water is 6 inches deep?

Section 3.11

12. Let \( y = 4 - x^2 \). Find \( \Delta y \) if \( x \) changes from \( x = 1 \) to \( x = 1.5 \).

13. If \( f(x) = 4 - x^2 \), find \( dy \) if \( x = 1 \) and \( dx = \frac{1}{2} \).

14. Find the differential \( dy \) if \( y = \frac{r}{r + 1} \) and \( dr = 0.5 \).

15. Use differentials to approximate \((1.97)^6\).

16. Use differentials to approximate \(\cos(31.5°)\).

17. Find the linear approximation for \( f(x) = \frac{1}{x} \) at \( x = 4 \).

18. Find the linear approximation for \( f(x) = \sqrt{x + 1} \) at \( x = 0 \) and use it to approximate \( \sqrt{0.95} \).
19. The radius of a circular disk is given to be 24 cm with a maximum error in measurement of 0.2 cm.

(a) What is the maximum error in the calculated area of the disk?

(b) Use differentials to approximate the maximum error in the area of the disk.

(c) What is the relative error?

20. Suppose for a function $f$, the linear approximation for $f(x)$ at $a = 3$ is given by $y = 2x + 7$.

a.) Find the value of $f'(3)$ and $f(3)$.

b.) If $g(x) = \sqrt{f(x)}$, find the linear approximation for $g(x)$ at $a = 3$.

21. Find the quadratic approximation for $f(x) = \cos x$ at $x = \frac{\pi}{3}$. 