

MATH 152
SPRING 2018

Sample Exam (covering sections 5.5-7.2)

1. Find the area of the region bounded by $y = x^3$, $y = x$ from $x = 0$ to $x = 2$.

a) $\frac{3}{2}$

b) 2

c) $\frac{1}{2}$

d) $\frac{5}{2}$

e) 3



$$\int_0^1 x - x^3 \, dx + \int_1^2 x^3 - x \, dx$$

$$\left[\frac{x^2}{2} - \frac{x^4}{4} + \left(\frac{x^4}{4} - \frac{x^2}{2} \right) \right]_1^2 = \frac{4}{4} + 4 - 2 - \frac{1}{2} + \frac{1}{2}$$

2. If we revolve the region bounded by $x = 2y^2$ and $x = 2$ about the line $x = 2$, which of the following integrals gives the resulting volume?

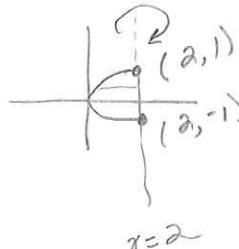
a) $\int_{-1}^1 \pi(4 - 4y^4) \, dy$

b) $\int_{-1}^1 \pi(4 - (2 - 2y^2)^2) \, dy$

c) $\int_{-1}^1 4\pi y^4 \, dy$

d) $\int_{-1}^1 \pi(2 - 2y^2)^2 \, dy$

e) $\int_{-1}^1 \pi(4y^4 - 4) \, dy$



$$\int_{-1}^1 \pi(2 - 2y^2)^2 \, dy$$

3. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

a) $\frac{75}{4}$ J

$$f(1) = 10 \quad k = 10$$

b) 45 J

c) $\frac{75}{2}$ J

d) 30 J

e) 40 J

$$\int_1^3 10x \, dx = 5x^2 \Big|_1^3$$

$$= 5(8)J$$

$$t = x^3$$

$$dt = 3x^2 dx$$

4. Evaluate $\int_0^{\sqrt[3]{\pi/2}} x^5 \cos(x^3) dx$

a) $\frac{\pi}{6} - \frac{1}{3}$
 b) $\frac{\pi}{3} - \frac{1}{6}$
 c) $\frac{\pi}{2} - \frac{1}{3}$
 d) $\frac{\pi}{3} - \frac{1}{2}$
 e) $\frac{\pi}{6} - \frac{1}{2}$

$\frac{1}{3} \int_0^{\frac{\pi}{2}} t \cos t dt$

u	dv
t	$\cos t$
0	$\sin t$
	$-\cos t$

$\frac{1}{3} \left(t \sin t + \cos t \right) \Big|_0^{\frac{\pi}{2}}$

$\frac{1}{3} \left(\frac{\pi}{2} - 1 \right) = \frac{\pi}{6} - \frac{1}{3}$

5. $\int_1^{e^4} x \ln x dx =$

a) $\frac{7e^8 + 1}{4}$
 b) $\frac{9e^8 + 1}{4}$
 c) $\frac{8e^8 + 1}{4}$
 d) $\frac{7e^8 - 1}{4}$
 e) $\frac{8e^8 - 1}{4}$

$u = \ln x \quad dv = x dx$
 $du = \frac{dx}{x} \quad v = \frac{x^2}{2}$

$\frac{x^2}{2} \ln x \Big|_1^{e^4} - \frac{1}{2} \int x dx$

$\frac{x^2}{2} \ln x - \frac{x^2}{4} \Big|_1^{e^4} = \frac{e^8}{2}(4) - \frac{e^8}{4} + \frac{1}{4}$

6. $\int \sin^2(x) dx =$

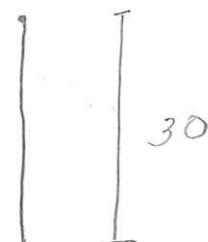
a) $\frac{x}{2} + \frac{1}{4} \sin(2x) + C$
 b) $\frac{x}{2} - \frac{1}{4} \sin(2x) + C$
 c) $\frac{4}{3} \sin^3(x) + C$
 d) $\frac{x}{2} + 2 \sin(2x) + C$
 e) $\frac{1}{3} \sin^3(x) + C$

$$\int \frac{1}{2} (1 - \cos 2x) dx$$

$$\frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + C$$

7. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling $\frac{1}{3}$ of this rope to the top of the cliff?

- a) 125 foot-pounds
- b) 25 foot-pounds
- c) 35 foot-pounds
- d) 2255 foot-pounds
- e) 75 foot-pounds



method 1 weighs $\frac{1}{2} \text{ lb/foot}$

$$W = \int_0^{10} \frac{1}{2}x dx + (20)(\frac{1}{2})(10)$$

$$= \frac{x^2}{4} \Big|_0^{10} + 100$$

$$= 125$$

method 2 $\int_0^{10} 15 - \frac{1}{2}x dx$

$$15x - \frac{1}{4}x^2 \Big|_0^{10}$$

$$= 150 - 25$$

8. $\int_0^{\pi/4} \sec^4 x \tan^2 x dx$

- a) $\frac{16}{3}$
- b) $\frac{4}{3}$
- c) $\frac{8}{3}$
- d) $\frac{1}{6}$
- e) $\frac{8}{15}$

$$\int_0^{\frac{\pi}{4}} \sec^2 x \tan^2 x \sec^2 x dx$$

$$\int_0^{\frac{\pi}{4}} (\tan^2 x + 1) \tan^2 x \sec^2 x dx$$

$$\int_0^1 (u^4 + u^2) du = \frac{1}{5} + \frac{1}{3} = \boxed{\frac{8}{15}}$$

9. $\int \frac{x}{(x-1)^2} dx$

- a) $\ln|x-1| + \frac{1}{x-1} + C$
- b) $\ln|x-1| - \frac{1}{x-1} + C$
- c) $\ln|x-1| + \frac{1}{3(x-1)^2} + C$
- d) $\ln|x-1| - \frac{1}{3(x-1)^2} + C$
- e) $\ln|x-1| + \frac{3}{(x-1)^2} + C$

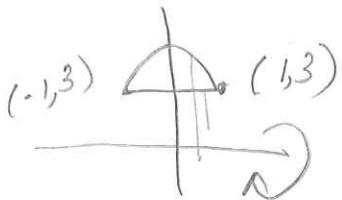
$$u = x-1 \quad x = u+1$$

$$\int \frac{u+1}{u^2} du = \int \frac{1}{u} + \frac{1}{u^2} du$$

$$= \boxed{\ln|x-1| - \frac{1}{x-1} + C}$$

Part II - Work Out Problems

10. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the x -axis.

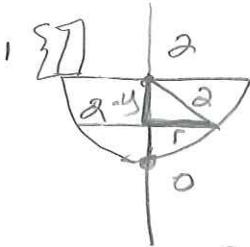


$$\begin{aligned}
 V &= 2\int_0^1 \pi((4-x^2)^2 - 9) dx \\
 &= 2\pi \int_0^1 (16 - 8x^2 + x^4) - 9 dx \\
 &= 2\pi \int_0^1 (7 - 8x^2 + x^4) dx \\
 &= 2\pi \left(7x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right) \Big|_0^1 \\
 &= 2\pi \left(7 - \frac{8}{3} + \frac{1}{5} \right)
 \end{aligned}$$

11. The base of a solid is the region bounded by $y = x^2$ and $y = 1$. Cross-sections perpendicular to the y -axis are semi-circles. Set up but do not evaluate an integral that gives the volume of the solid.

$$\begin{aligned}
 &\int_0^1 \frac{1}{2}\pi r^2 dy \\
 &\int_0^1 \frac{\pi}{2}(y) dy
 \end{aligned}$$

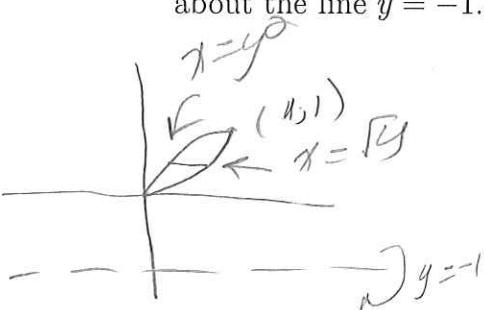
12. A 15 m long trough with semicircular ends of radius 2 m is full of water. Set up but do not evaluate an integral that will compute the work required to pump all of the water out of a 1 m high spout. Indicate on the picture where you are placing the axis and which direction is positive. Note: The density of water is $\rho = 1000 \text{ kg/m}^3$ and the acceleration due to gravity is 9.8 m/s^2 .



$$W = \int_0^2 \pi \rho g (4y - y^2)(2 - y + 1) dy$$

$$\begin{aligned} 4 &= (2-y)^2 + r^2 \\ r^2 &= 4 - (2-y)^2 \\ &= 4 - (4 - 4y + y^2) \\ r^2 &= 4y - y^2 \end{aligned}$$

13. Using cylindrical shells, set up but do not evaluate an integral that gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.

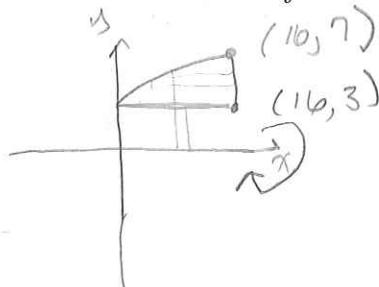


$$V = \int_0^1 2\pi(y+1)(\sqrt{y} - y^2) dy$$

$$x = (y-3)^2$$

14. Consider the region R bounded by $y = \sqrt{x} + 3$, $y = 3$, $x = 16$

a.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the x -axis



$$V = \int_0^{16} \pi \left((\sqrt{x} + 3)^2 - 9 \right) dx \quad \text{or}$$

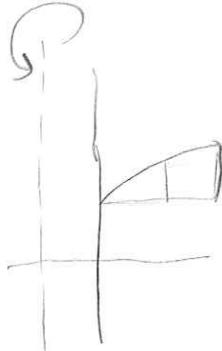
$$\int_3^7 2\pi y (16 - (y-3)^2) dy$$

b.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the y -axis

$$\int_0^{16} 2\pi x (\sqrt{x} + 3 - 3) dx \quad \text{or}$$

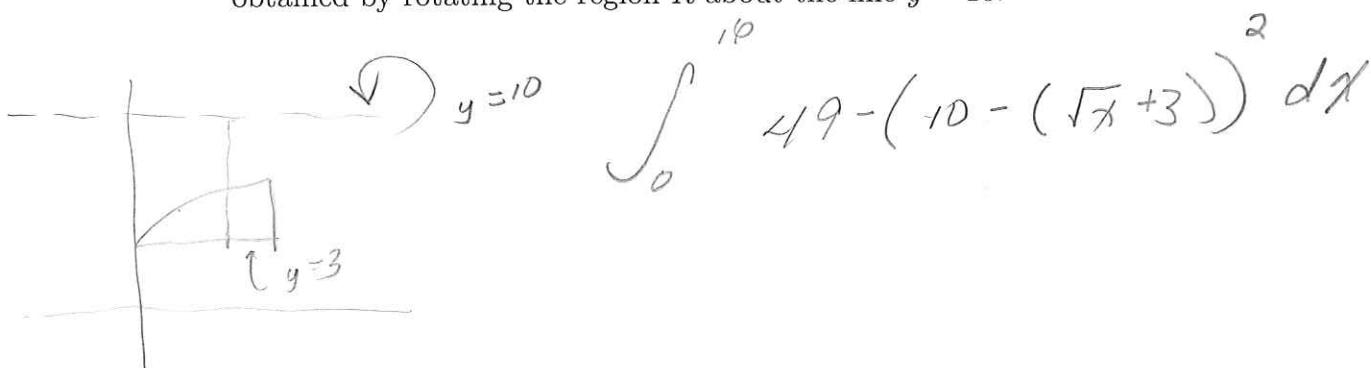
$$\int_3^7 \pi (16^2 - (y-3)^2) dy$$

c.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $x = -1$



$$x = -1$$

d.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region R about the line $y = 10$.



$$\begin{aligned}
 15. \text{ Find } \int \sec^5 x \tan^3 x dx. &= \int \sec^4 x \tan^2 x \sec x \tan x dx \\
 &= \int \sec^4 x (\sec^2 x - 1) \sec x \tan x dx \\
 u = \sec x &= \int u^4 (u^2 - 1) du \\
 du = \sec x \tan x dx &= \int u^4 - u^6 du = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C
 \end{aligned}$$

$$\begin{aligned}
 16. \text{ Find } \int \sin^5(3x) \cos^2(3x) dx. &= \int \sin^4(3x) \cos^2(3x) \sin(3x) dx \\
 u = \cos 3x &= (\sin^2(3x))^2 \cos^2(3x) \sin(3x) dx \\
 du = -3 \sin(3x) dx &= (1 - \cos^2(3x))^2 \cos^2(3x) \sin(3x) dx \\
 &= -\frac{1}{3} \int (1 - u^2)^2 u^2 du
 \end{aligned}$$

$$\begin{aligned}
 17. \text{ Evaluate } \int \arccos x dx. \quad u = \arccos x \quad du = dx \\
 du = -\frac{1}{\sqrt{1-x^2}} dx \quad v = x
 \end{aligned}$$

$$\arccos x + \int \frac{x}{\sqrt{1-x^2}} dx \quad u = 1-x^2 \quad u^{-5/2} v^2$$

$$\arccos x - \sqrt{1-x^2} + C$$

17. Evaluate $\int e^x \cos(2x) dx$.

$$u = \cos(2x) \quad dv = e^x dx$$

$$du = -2\sin(2x)dx \quad v = e^x$$

$$\int e^x \cos(2x) dx = e^x \cos(2x) + \underbrace{\int 2e^x \sin(2x) dx}$$

$$\downarrow$$

$$u = 2\sin(2x) \quad dv = e^x dx$$

$$du = 4\cos(2x)dx \quad v = e^x$$

$$= e^x \cos(2x) + 2e^x \sin(2x) - \int 4e^x \cos(2x) dx$$

$$5 \int e^x \cos(2x) dx = e^x \cos(2x) + 2e^x \sin(2x)$$

$$\int e^x \cos(2x) dx = \frac{e^x \cos(2x) + 2e^x \sin(2x)}{5} + C$$