MATH 151, FALL 2015
SAMPLE EXAM II

PART I: Multiple Choice: 3 points each

1. Find the derivative of $f(x) = \sin(\tan x)$

   (a) $(\tan x)(\sec x)\cos(\tan x)$

   (b) $(\sec^2 x)\cos(\tan x)$

   (c) $-(\sec^2 x)\cos(\tan x)$

   (d) $\sin(\sec^2 x)$

   (e) $(\sec x)\cos(\tan x)$

2. $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{\sin 2\theta} =$

   (a) $\frac{3}{2}$

   (b) $\frac{2}{3}$

   (c) 1

   (d) 0

   (e) ∞

3. If $xe^y = 4$, find $\dfrac{dy}{dt}$ when $y = 0$ given that $\dfrac{dx}{dt} = 10$.

   (a) $-\frac{1}{4}$

   (b) $-\frac{5}{2}$

   (c) $\frac{5}{2}$

   (d) $\frac{1}{8}$

   (e) $\frac{1}{4}$

4. $\lim_{x \to 4^-} \left(\frac{1}{e}\right)^{\frac{1}{x-4}} =$

(a) e

(b) 0

(c) $\frac{1}{e}$

(d) $-\infty$

(e) ∞

5. Consider the parametric curve $x(t) = t^4 + 1$ and $y(t) = \cos\left(\frac{\pi}{2}t\right)$. Find the slope of the tangent line at the point $(2, 0)$.

(a) $-\frac{\pi}{8}$

(b) $-\frac{1}{4}$

(c) $-\frac{8}{\pi}$

(d) 0

(e) $-\frac{\pi}{4}$

6. If $f(x) = x\, g(\sqrt{x})$, then $f'(x) =$

(a) $\frac{1}{2}\sqrt{x}\, g'(\sqrt{x})$

(b) $\frac{1}{2}\sqrt{x}\, g'(\sqrt{x}) - g(\sqrt{x})$

(c) $-\frac{1}{2}\sqrt{x}\, g'(\sqrt{x}) + g(\sqrt{x})$

(d) $\frac{1}{2}\sqrt{x}\, g'(\sqrt{x}) + g(\sqrt{x})$

(e) $g(\sqrt{x})$

7. The graph of the curve $y = x + \frac{1}{3}\cos(3x)$ has a horizontal tangent at $x =$

(a) $\frac{\pi}{12}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

(e) π

8. If $f(x) = e^{\tan x}$, then $f''\left(\frac{\pi}{4}\right) =$

(a) $4e$

(b) $2\sqrt{2}e$

(c) $6e$

(d) $\sqrt{2}e$

(e) $8e$

9. Find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2}, \pi\right)$ for $x\sin(y) - y\cos(2x) = 2x$.

(a) $\frac{4}{2+\pi}$

(b) $\frac{4\pi}{2-\pi}$

(c) $\frac{4\pi}{2+\pi}$

(d) $\frac{4}{2-\pi}$

(e) $\frac{2}{2+\pi}$

10. Find the point of intersection of the curves $\mathbf{r_1}(t) = \langle 1-t, 3+t^2 \rangle$ and $\mathbf{r_2}(w) = \langle w-2, w^2 \rangle$.

(a) $(1,2)$

(b) $(0,2)$

(c) $(0,4)$

(d) $(1,1)$

(e) $(-1,1)$

11. At what point on the curve $y = \dfrac{1}{(x+1)^2}$ is the tangent line parallel to $4y = x+8$?

(a) $\left(-6, \dfrac{1}{25}\right)$

(b) $\left(1, \dfrac{1}{4}\right)$

(c) $\left(-3, \dfrac{1}{4}\right)$

(d) $\left(-9, \dfrac{1}{64}\right)$

(e) $\left(3, \dfrac{1}{16}\right)$

12. Find the equation of the tangent line to the curve $x = 1-t$, $y = 2-t^2$ at the point $(2,1)$.

(a) $y = 4x - 7$

(b) $y = 2x - 3$

(c) $y = -2x + 5$

(d) $y = -x + 3$

(e) $y = -4x + 7$

13. A particle moves according to the equation of motion $s(t) = 2t^3 - 9t^2 + 12t + 5$, where t is measured in seconds and $s(t)$ is measured in feet. When is the particle moving in the positive direction?

(a) $(1, 2)$

(b) $(-\infty, 1) \cup (2, \infty)$

(c) $(2, \infty)$

(d) $[0, 1) \cup (2, \infty)$

(e) $(0, 1) \cup (2, \infty)$

14. Find the quadratic approximation for $f(x) = \sqrt[3]{x+1}$ at $x = 7$.

(a) $Q(x) = 2 + \frac{1}{12}(x-7) - \frac{1}{144}(x-7)^2$

(b) $Q(x) = 2 + \frac{1}{12}(x-7) - \frac{1}{288}(x-7)^2$

(c) $Q(x) = 2 + \frac{1}{12}(x-7) + \frac{1}{288}(x-7)^2$

(d) $Q(x) = 2 + \frac{1}{12}(x-7) + \frac{1}{144}(x-7)^2$

(e) $Q(x) = 2 + \frac{1}{12}(x-7) + \frac{1}{72}(x-7)^2$

15. If $\mathbf{r(t)} = \langle 2\cos t, 3\sin t \rangle$. Describe the direction of the curve as t increases.

(a) Clockwise around the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(b) Counterclockwise around the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(c) Clockwise around the curve $(x-2)^2 + (y-3)^2 = 1$.

(d) Counterclockwise around the curve $(x-2)^2 + (y-3)^2 = 1$.

(e) Clockwise around the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Consider the function $g(x) = \sin(x)$.

a.) (6 pts) Find the linear approximation of $g(x)$ for values of x near 30°.

b.) (5 pts) Use your answer above to approximate $\sin(29^\circ)$.

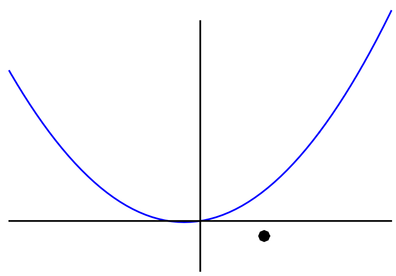
17. Consider $x = \sin(2t)$, $y = 2\cos(2t) + \sin(2t)$.

(a) (6 pts) Find the equation of the tangent line to the curve given by the parametric equations at the point on the curve where $t = \pi/8$.

(b) (5 pts) Find *any* point on the curve were the tangent line is vertical. Support your answer.

18. Suppose we have a circular cone with height 1 foot and radius 3 feet, vertex at the bottom. Suppose further that a solution is being poured into the cone at a rate of 0.5 cubic feet per minute. How fast is the height of the solution increasing when the volume of the solution is $\frac{3\pi}{8}$ cubic feet?

19. (a) (3 pts) Show by means of a sketch that there are two lines tangent to the parabola $y = x^2 + x$ that pass through the point $(2, -3)$.



(b) (8 pts) Find an equation of each of these tangent lines.

20. Consider $f(x) = \begin{cases} ax^2 + 4cx + 3 & \text{if } x \leq -1 \\ cx - 6 & \text{if } x > -1 \end{cases}$

(a) (8 pts) Find the values of a and c that make $f(x)$ differentiable everywhere.

(b) (3 pts) For the values of a and c found above, find $f'(x)$

21. Suppose g is the inverse of f and $f(2) = 3$, $f'(2) = 7$, $f(3) = 4$ and $f'(3) = \frac{1}{2}$. Find $g'(3)$.

22. Suppose g is the inverse of f. Find $g'(4)$ if $f(x) = 3 + x + e^x$.