MATH 151, SUMMER 2016
SAMPLE EXAM III

PART I: Multiple Choice: 3 points each

1. Given $f(x) = x^3 \ln x$, find $f'(e)$

   (a) e

   (b) $3 + 3e^2$

   (c) e^2

   (d) $3e$

   (e) $4e^2$

2. $\dfrac{d}{dx}(\tan^{-1}(x^2)) =$

   (a) $\dfrac{2x}{1+x^4}$

   (b) $\dfrac{2}{1+x^2}$

   (c) $-2x\csc(x^2)\cot(x^2)$

   (d) $2x\tan^{-1}(x^2)\sec^{-1}(x^2)$

   (e) $\dfrac{2x}{1+x^2}$

3. Solve the equation $\ln x + \ln(x+1) = \ln(x+4)$ for x.

   (a) $x = 0$ and $x = 3$

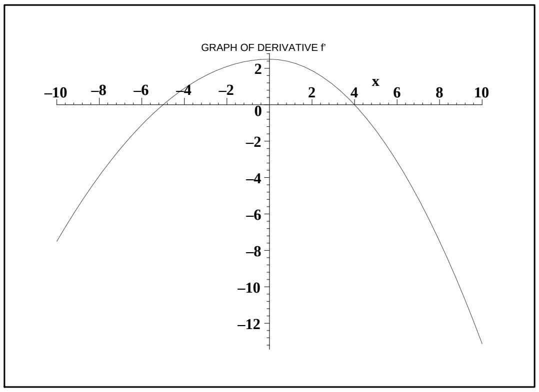
   (b) $x = 4$

   (c) $x = \pm 2$

   (d) $x = 3$

   (e) $x = 2$

4. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function f concave down?
CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!



(a) $(-\infty, -5)$

(b) $(-5, 0)$

(c) $(0, 4)$

(d) $(4, \infty)$

(e) none of these intervals

5. Circle ALL the critical values of $f(x) = x(x-1)^{\frac{1}{3}}$
NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

(a) 0

(b) $-\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{4}$

(e) 1

6. Find the absolute maximum of $f(x) = \sin x + \cos x$ on the interval $\left[0, \frac{\pi}{3}\right]$.
(**NOTE**: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.73$)

(a) 1

(b) 2

(c) $\sqrt{2}$

(d) $\frac{\sqrt{3}+1}{2}$

(e) $\frac{\pi}{4}$

7. The inflection points of $f(x) = x^5 + 10x^4$ occur at which of the following?

(a) $x = 6$ only

(b) $x = -6$ only

(c) $x = 0$, $x = -8$

(d) $x = 0$, $x = -6$

(e) $x = 0$, $x = 6$

8. Which is an antiderivative of $f(x) = 2\sqrt{x} + \frac{1}{x^2}$?

(a) $\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C$

(b) $\frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

(c) $\frac{1}{\sqrt{x}} - \frac{2}{x^3} + C$

(d) $\frac{1}{\sqrt{x}} + \ln(x^2) + C$

(e) $\frac{1}{\sqrt{x}} - \frac{1}{x} + C$

9. Given $x = 2$ is a critical number for $f(x) = x^3 e^{-bx}$, what is b?

(a) $\frac{3}{2}$

(b) 8

(c) $-\frac{1}{2}\ln 8$

(d) $\frac{2}{3}$

(e) 12

10. $\tan\left(\arccos\left(\frac{x}{2}\right)\right) =$

(a) $\dfrac{x}{\sqrt{x^2+4}}$

(b) $-\dfrac{1}{\sqrt{4-x^2}}$

(c) $\dfrac{\sqrt{4-x^2}}{x}$

(d) $\dfrac{2}{x^2+4}$

(e) $\dfrac{x}{\sqrt{4-x^2}}$

11. Find $\lim_{x\to 0} \dfrac{\arctan x - x}{x^3}$

(a) $-\dfrac{1}{3}$

(b) $-\dfrac{1}{2}$

(c) 0

(d) ∞

(e) 6

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

12. Find the derivative of

(i) $f(x) = x^{\sec x}$

(ii) $2^{\arcsin x} + \log_2(x^2)$

13. Find $\lim_{x \to 0} (1 - x)^{\frac{5}{x}}$

14. For $f(x) = x^2 \ln(x)$:

   a.) Find the domain of $f(x)$.

   b.) Find $\lim_{x \to 0^+} f(x)$.

   c.) Find the intervals where $f(x)$ is increasing and decreasing and find all local extrema of $f(x)$

15. Find the intervals of concavity and inflection point(s) for $f(x) = xe^{4x}$

16. If 1200 square cm of material is available to make a box with a square base, find the largest possible volume of the box.

17. The acceleration of a particle is given by $\mathbf{a}(t) = (1 + e^t)\mathbf{i} + (\cos\, t)\mathbf{j}$. If the initial velocity is $\mathbf{i}$ and the initial position is $\mathbf{j}$, find the position of the particle at any time t.

18. A thermometer is taken from a room where the temperature is 75° to the outdoors, where the temperature is 35°. After one minute, the thermometer reads 60°. What is the reading of the thermometer at time t?

19. A bacterial culture starts with 200 bacteria and triples in size every half hour. Assuming exponential growth, how many bacteria are there after 45 minutes?

20. Find the following:

a.) $\arccos\left(\frac{1}{2}\right) =$ ______

b.) $\sin\left(\arccos\left(-\frac{4}{5}\right)\right) =$ ______

c.) $\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right) =$ ______

d.) The domain of $\arcsin(4x - 5) =$ ____________

21. You are given a function f, an interval, partition points, and a description of the point x_i^* within the ith subinterval.

(a) Find $||P||$.

(b) Sketch the graph of f and the approximating rectangles.

(c) Find the sum of the approximating rectangles.

$f(x) = 16 - x^2$, $[0, 4]$, $P = \{0, 1, 2, 3, 4\}$, $x_i^* =$ left endpoint.