Summer 2016 Math 151

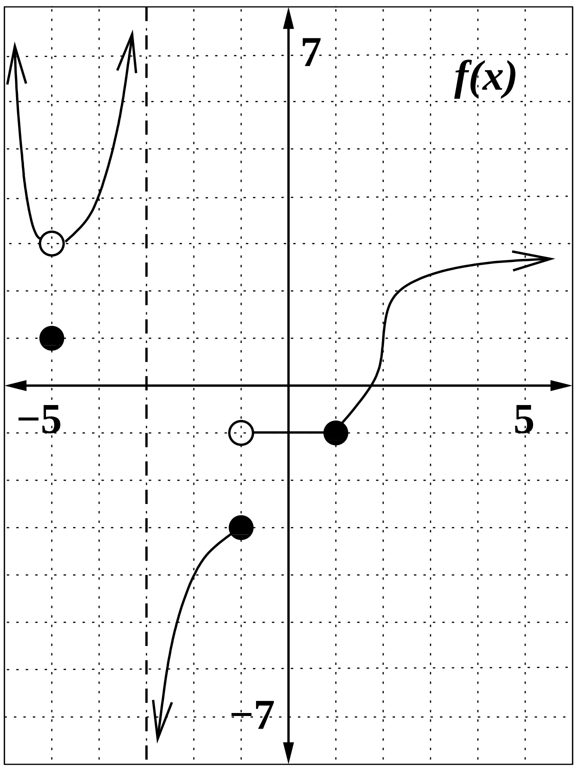
Week in Review 1

courtesy: Amy Austin

(covering 2.2-2.5)

Section 2.2

1. Consider the graph given below to find the indicated limits and/or values.



$\lim_{x\to -1^-} f(x) =$______, $\lim_{x\to -1^+} f(x) =$______,

$\lim_{x\to -1} f(x) =$______, $\lim_{x\to 1} f(x) =$______,

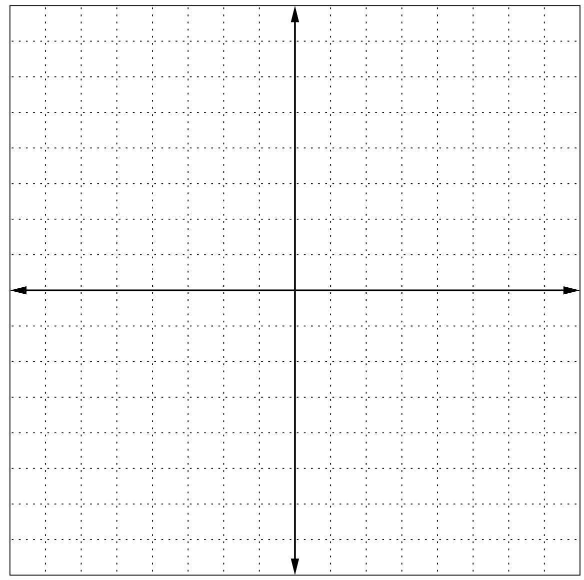
$\lim_{x\to -5} f(x) =$______, $\lim_{x\to \infty} f(x) =$______,

$f(-5)$______, $f(-1)$______ $f(-3)$______

$\lim_{x\to -3^+} f(x) =$______, $\lim_{x\to -3^-} f(x) =$______,

2. Consider $f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 4 & \text{if } x = 1 \\ 4-x & \text{if } x > 1 \end{cases}$

   Sketch the graph of $f(x)$ and find all values of x for which the limit does not exist.



3. $\lim_{x\to -2^+} \dfrac{x-1}{x+2}$

4. $\lim_{x\to 3^-} \dfrac{x-5}{x^2-9}$

5. $\lim_{x\to -2} \dfrac{x-1}{x^2(x+2)}$

6. Find all holes and vertical asymptote(s) on the graph of the function $f(x) = \dfrac{x-1}{x^2-1}$.

Section 2.3

Compute the exact value of the following limits. If the limit does not exist, support your answer by evaluating left and right hand limits.

7. $\lim_{x\to 1}(4x^3 - 3x + 1)$

8. $\lim_{x\to -5} \dfrac{x^2+5x}{x+5}$

9. $\lim_{x\to 2} \dfrac{x-\sqrt{3x-2}}{x^2-4}$

10. $\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$

11. $\lim_{x \to 1} \frac{x-4}{x-1}$

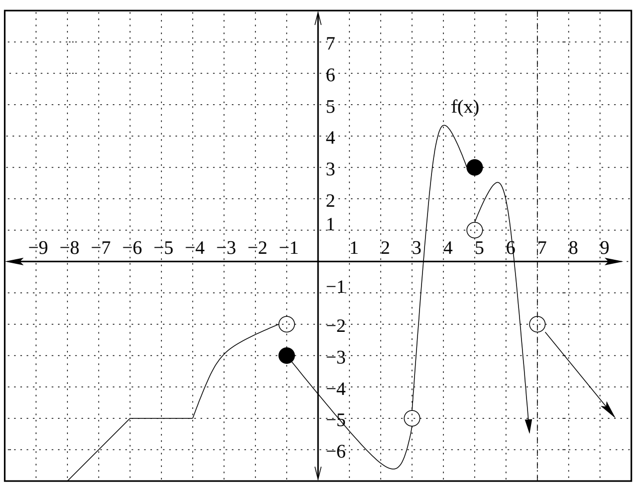
12. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} x+5 & \text{if } x \leq 3 \\ x^3 - 3 & \text{if } x > 3 \end{cases}$

13. $\lim_{x \to 2} \frac{x^2 - 4}{|\ x - 2\ |}$

14. $\lim_{x \to 1} f(x)$ if it is known that $4x \leq f(x) \leq x + 3$ for all x in $[0, 2]$.

Section 2.5

15. Referring to the graph, explain why the function $f(x)$ is or is not continuous (support your conclusion) at $x = -1$, $x = 3$, $x = 5$, $x = -4$ and $x = 7$.



16. If $f(x) = \frac{x+2}{x^2 + 5x + 6}$, find all values of $x = a$ where the function is discontinuous. For each discontinuity, find the limit as x approaches a, if the limit exists. If the limit does not exist, support your answer by evaluating left and right hand limits.

17. Suppose it is known that $f(x)$ is a continuous function defined on the interval $[1, 5]$. Suppose further it is given that $f(1) = -3$ and $f(5) = 6$. Give a graphical arguement that there is at least one solution to the equation $f(x) = 1$.

18. If $g(x) = x^5 - 2x^3 + x^2 + 2$, use the Intermediate Value Theorem to find an interval which contains a root of $g(x)$, that is contains a solution to the equation $g(x) = 0$.

19. Find the values of c and d that will make

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$$

continuous on all real numbers. Once the value of c and d is found, find $\lim_{x \to 1} f(x)$ and $\lim_{x \to 2} f(x)$.

Section 2.6

20. Compute the following limits:

a.) $\lim_{x \to \infty} \frac{4x^3 - 6x^4}{2x^3 - 9x + 1}$

b.) $\lim_{t \to -\infty} \frac{t^9 - 4t^{10}}{t^{12} + 2t^2 + 1}$

c.) $\lim_{x \to \infty} \frac{4x - 6x^3}{-2x^3 - 9x + 1}$

d.) $\lim_{x \to \infty} \frac{\sqrt{2 + x^2}}{4 - 7x}$

e.) $\lim_{x \to -\infty} \frac{\sqrt{5x^2 + 1}}{x - 3}$

f.) $\lim_{x \to \infty} (\sqrt{x^2 + 5x + 1} - x)$

g.) $\lim_{x \to -\infty} (x + \sqrt{x^2 + x + 2})$

21. Find all horizontal and vertical asymptotes of $f(x) = \frac{x^3}{x^3 - x}$