Summer 2016 Math 151

Week in Review 2

courtesy: Amy Austin

(covering 3.2-3.5)

Section 3.2

(1) Constant rule: If $f(x) = c$, where c is a constant, then $f'(x) = 0$.

(2) Power rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

(3) Constant times a function rule:

$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$

(4) Sum/Difference rule: If $f(x) = g(x) \pm h(x)$, then $f'(x) = g'(x) \pm h'(x)$

(5) Product rule: If $f(x) = g(x)h(x)$, then

$f'(x) = g(x)h'(x) + g'(x)h(x)$

(6) Quotient rule: If $f(x) = \frac{g(x)}{h(x)}$, then

$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$

1. Find the derivative of the following functions.

   (a) $g(x) = x^5 + 8x^2 - 16x + 2 - \pi^2$

   (b) $f(t) = (1 - \sqrt{t})^2$

   (c) $H(s) = \left(\frac{s}{2}\right)^5$

   (d) $F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$

   (e) $y = (x^3 - x^2 - 2x + 1)(5x^4 - 20x^3 + 5x + 3)$

   (f) $f(u) = \frac{1 - u^2}{1 + u^2}$

2. If $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$, find the value of $(fg)'(5)$.

3. Find the equation of the tangent line to the graph of $f(x) = x + \sqrt{x}$ at the point $(1, 2)$.

4. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

5. Show there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the equation of these tangent lines.

6. If $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$, find the value of m and b that make $f(x)$ differentiable everywhere.

7. If $\vec{\mathbf{r}}(t) = \langle t^2 + 2t, t^3 + 3t^2 \rangle$ is the position of a moving object at time t, where the position is measured in feet and the time in seconds, find the velocity and speed at time $t = 1$

Section 3.4

8. Find the limit:

   (a) $\lim_{x \to 0} \frac{\sin x}{3x}$

   (b) $\lim_{x \to 0} \frac{\sin 9x}{7x}$

   (c) $\lim_{x \to 0} \frac{\sin 8x}{\sin 7x}$

   (d) $\lim_{x \to 0} \frac{\tan^2 4x}{x^2}$

   (e) $\lim_{x \to 0} \frac{\cos x - 1}{\sin x}$

Derivatives of Trigonometric Functions:

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

9. Find the derivative:

   (a) $f(x) = \sec^2 x + 4\tan x + x\sqrt{x}$

   (b) $g(t) = \frac{2\cos t + 1}{\cot t + t}$

10. Find the equation of the tangent line to the graph of $f(x) = 2\sin x$ at $x = \frac{\pi}{3}$.

Section 3.5

Chain Rule: We use the chain rule when we are differentiating a function written as a composition of functions, that is $f(x) = g(h(x))$. Then $f'(x) = g'(h(x))h'(x)$.

11. Find the derivative:

    (a) $f(x) = \sin(2x) + \cot(5x^2)$

    (b) $g(t) = \tan(\cos(t))$

    (c) $h(w) = \sec(\cos(\sin(4w^2)))$

Generalized Power Rule: If $f(x) = (g(x))^n$, then $f'(x) = n\,(g(x))^{n-1}\,g'(x)$

12. Find the derivative:

    (a) $f(x) = \dfrac{1}{(x^2 + 5x + 4)^{10}}$

    (b) $g(x) = x^3(\sqrt{x} + 5)^3$

    (c) $f(x) = \sin(3x) + \sin^3(x)$

    (d) $h(t) = \sqrt{\cos(\sin^2 t)}$

13. Find the equation of the tangent line to the graph of $f(x) = 8\sqrt{4 + 3x}$ at $x = 4$.

14. Suppose $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$ and $v'(2) = 6$. Find $w'(0)$.

15. If $F(x) = f(\cos x)$, $G(x) = \cos(f(x))$ and $H(x) = [f(\sin x)]^3$, find $F'(x)$ and $G'(x)$ and $H'(x)$.