Summer 2016 Math 151

Week in Review 4

courtesy: Amy Austin

(covering 4.8-5.5)

Section 4.8

1. Find the following limits.

   a.) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x-1}$

   b.) $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

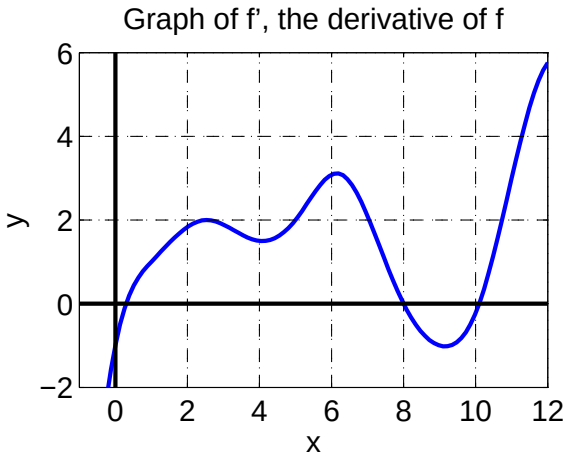
   c.) $\lim_{x\to 0^+} x^2 \ln x$

   d.) $\lim_{x\to\infty} (e^x + x)^{\frac{1}{x}}$

   e.) $\lim_{x\to 0} (\sin x)^{\tan x}$

Section 5.1

2. Given the graph of $f'(x)$ find intervals if increase/decrease, local extrema, intervals of concavity and inflection points.



3. Sketch a graph satisfying:

   a.) Domain: All real numbers

   b.) $f(-1) = -2$, $f(0) = 0$, $f(2) = 3$

   c.) $f'(x) < 0$ for $x < -1$ and $x > 2$

   d.) $f'(x) > 0$ if $-1 < x < 2$

   e.) $f''(x) > 0$ if $x < 0$ and $f''(x) < 0$ if $x > 0$

Section 5.2

4. For the following functions, identify all critical values.

   a.) $f(x) = 4x^3 - 9x^2 - 12x + 3$

   b.) $f(x) = x^2 e^{2x}$

   c.) $f(x) = |x^2 - 2x|$

   d.) $f(x) = (x^2 - x)^{1/3}$

   e.) $f(x) = \dfrac{x+1}{x-2}$

5. Find the absolute and local extrema for the following functions by graphing.

   a.) $f(x) = 1 - x^2$, $-1 < x \leq 2$

   b.) $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ 2 - x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$

6. Find the absolute extrema for $f(x) = x^3 - 12x + 1$ over the interval $[-1, 5]$

Section 5.3

7. State the Mean Value Theorem. Verify $f(x) = x^2$ satisfies the Mean Value Theorem on the interval $[-1, 2]$. Find all c that satisfies the conclusion of the Mean Value Theorem.

8. Find the intervals where the given function is increasing or decreasing and identify all local extrema:

   a) $f(x) = 3x^4 + 4x^3 - 12x^2 + 8$

   b) $f(x) = \dfrac{x}{(x-1)^2}$

   c) $f(x) = (x^2 - x)^{1/3}$

9. Determine the intervals where the given function is concave up or concave down and identify all inflection points for $f(x) = x^5 + 5x^4$

10. Given $f(-3) = 4$, $f'(-3) = 0$, $f''(-3) = 7$, $f(2) = -5$, $f'(2) = 0$, and $f''(2) = -6$, identify any local extrema of f.

Section 5.5

11. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

12. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

13. A piece of wire 12 inches long is cut into two pieces. One piece is bent into an equilateral triangle and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a maximum? A minimum?

14. What are the dimensions of the largest rectangle that can be inscribed in the area bounded by the curve $y = 12 - x^2$ and the x-axis?