MATH 152
SUMMER 2016

SAMPLE EXAM I (covering sections 6.5-8.9)

1. Find the area of the region (in the first quadrant) bounded by the curves $y = 5 - x^2$, $y = 4x$, $x = 0$, $x = 2$.

   a) $\frac{2}{3}$

   b) 3

   c) $\frac{16}{3}$

   d) $\frac{32}{5}$

   e) 6

2. If we revolve the region bounded by $x = (y-1)^2$ and $x = y + 1$ about the y-axis, which of the following integrals gives the resulting volume?

   a) $\int_0^3 2\pi y \left((y+1) - (y-1)^2\right)\, dy$

   b) $\int_0^1 2\pi y \left((y+1) - (y-1)^2\right)\, dy$

   c) $\int_0^3 2\pi (3-y) \left((y+1) - (y-1)^2\right)\, dy$

   d) $\int_0^3 \left(\pi (y+1)^2 - \pi (y-1)^4\right)\, dy$

   e) None of the above

3. Find the volume of the solid formed by rotating the region bounded by $y = \sin x$, $y = 0$ and $x = \frac{\pi}{2}$ about the x-axis.

   a) $2\pi^2$

   b) $\frac{\pi^2}{2} - 1$

   c) $\frac{\pi^2}{4} - 1$

   d) $\frac{\pi^2}{4}$

   e) $\frac{\pi^2}{2}$

4. Find the volume of the solid formed by rotating the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$ about the line $x = 3$.

a) $3\pi e - 4\pi$

b) $\dfrac{29\pi}{2} + \dfrac{1}{2}\pi e^2 - 6\pi e$

c) $6\pi e - 8\pi$

d) $\dfrac{29\pi}{4} + \dfrac{1}{4}\pi e^2 - 3\pi e$

e) $-2\pi + 2\pi e$

5. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.

a) $\dfrac{75}{4}$ J

b) 45 J

c) $\dfrac{75}{2}$ J

d) 30 J

e) 40 J

6. For what value of b is the average value of $f(x) = \sqrt{x}$ on the interval $[0, b]$ equal to 4?

a) $b = 8$

b) $b = 12$

c) $b = 36$

d) $b = 9$

e) None of these

7. $\int_1^{e^4} x \ln x \, dx =$

a) $\dfrac{7e^8 + 1}{4}$

b) $\dfrac{9e^8 + 1}{4}$

c) $\dfrac{8e^8 + 1}{4}$

d) $\dfrac{7e^8 - 1}{4}$

e) $\dfrac{8e^8 - 1}{4}$

8. $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}} \, dx =$

a) $\dfrac{1}{2}\sqrt{x^2+1} + C$

b) $\dfrac{1}{3}(x^2+1)^{3/2} - \sqrt{x^2+1} + C$

c) $\dfrac{1}{2}(x^2+1)^{3/2} - 2\sqrt{x^2+1} + C$

d) $\dfrac{2}{3}\sqrt{x^2+1} + C$

e) $\dfrac{3}{4}(x^2+1)^{3/2} - \sqrt{x^2+1} + C$

9. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling $\dfrac{1}{3}$ of this rope to the top of the cliff?

a) 125 foot-pounds

b) 25 foot-pounds

c) 35 foot-pounds

d) 2255 foot-pounds

e) 75 foot-pounds

10. Find the area the between the curves $x = 1 - y^2$ and $x = y^2 - 1$.

a) $\frac{1}{3}$

b) $\frac{4}{3}$

c) $\frac{5}{3}$

d) $\frac{2}{3}$

e) $\frac{8}{3}$

11. Find the average value of $f(x) = \sec^4 x$ over the interval $\left[0, \frac{\pi}{4}\right]$.

a) $\frac{\pi}{3}$

b) $\frac{16}{3\pi}$

c) $\frac{8}{3\pi}$

d) $\frac{\pi}{6}$

e) None of these

12. The integral $\int_1^\infty \frac{\sin^2 x}{x^2}\, dx$

a) converges to 0

b) diverges to ∞

c) diverges by oscillation

d) converges by comparison to $\int_1^\infty \frac{1}{x^2}\, dx$

e) diverges by comparison to $\int_1^\infty \frac{1}{x}\, dx$

13. $\int_0^\infty xe^{-x^2/2}\,dx =$

a) divergent

b) $\frac{1}{2}$

c) 0

d) $\frac{1}{4}$

e) 1

14. Find the length of the curve $y = 4x^{3/2}$ from $(0, 0)$ to $(2, 4)$.

a) $\frac{1}{54}(73\sqrt{73} - 1)$

b) $\frac{1}{27}(73\sqrt{73} - 1)$

c) $\frac{1}{54}(37\sqrt{37} - 1)$

d) $\frac{1}{27}(37\sqrt{37} - 1)$

e) None of these.

Part II - Work Out Problems

15. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the x-axis.

16. The base of a solid is the triangle with vertices $(0, 0)$, $(0, 4)$, and $(2, 0)$. Cross-sections perpendicular the the x-axis are semicircles. Find the volume of the solid.

17. A 15 foot long trough with semicircular ends of radius 2 ft is full of water. Find the work done in pumping the water out of the tank. The weight density of water is $\rho g = 62.5$ pounds per cubic foot.

18. Consider the region bounded by $y = x^2$ and $y = 2x$. Find the volume obtained by revolving this region about the line $x = 3$.

19. Consider the region R bounded by $y = \tan x$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{4}$.

a.) Sketch the region R.

b.) Find the area of this region.

c.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region about the line $y = -1$.

d.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region about the line $x = \dfrac{\pi}{2}$.

20. Find $\int \sec^5 x \tan^3 x \, dx$.

21. Evaluate $\int_0^{1/2} \arccos x \, dx$.

22. Find $\int e^x \sin x \, dx$

23. Find the surface area obtained by rotating the curve parametrized by $x(t) = \cos^2 t,\ y(t) = \sin^2 t,\ 0 \leq t \leq \dfrac{\pi}{2}$ about the y axis.

24. Integrate: $\int \frac{x^4+1}{x^2-4}\,dx$

25. $\int_{-2}^{-1} \frac{dx}{\sqrt{x^2+4x+5}} =$

26. The integral $\int_0^3 \frac{dx}{4x-1}$

27. Integrate: $\int \frac{2x^2 - x + 4}{x^3 + 4x}\, dx$

28. Find the surface area obtained by rotating the curve $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$, $1 \leq x \leq 3$, about the x axis.

$x(t) = \cos^2 t,\ y(t) = \sin^2 t,\ 0 \leq t \leq \dfrac{\pi}{2}$ about the y axis.

29. $\displaystyle\int \frac{dx}{x^2\sqrt{1-x^2}} =$